The Framework Theory Approach to Conceptual Change

Stella Vosniadou, Xenia Vamvakoussi and Irini Skopeliti

National and Kapodistrian University of Athens

Please address correspondence to
Stella Vosniadou
Department of Philosophy and History of Science
University of Athens
University Town, Athens, Greece
e-mail: svosniad@phs.uoa.gr

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Introduction

The term 'conceptual change' was introduced by Thomas Kuhn (1962) to indicate that the concepts embedded in a scientific theory change their meaning when the theory (paradigm) changes[1]. Kuhn promoted a contextual view of concepts as having an internal structure embedded in theoretical frameworks from which they obtain their meaning. When a theoretical framework changes, the meanings of the concepts subsumed in it also changes, making them 'incommensurable' to the same concepts subsumed under the previous theoretical framework[2]. The notion of incommensurability received considerable criticism from philosophers and historians of science, forcing Kuhn to eventually change his position from that of 'global incommensurability' to 'local incommensurability'. Local incommensurability refers to only a partial change in the meaning of concepts.

Susan Carey (see, Carey, 1985, 1991, and Carey & Spelke, 1994) was instrumental in clarifying how conceptual change can be seen in the context of cognitive development. Carey (1991) supported the notion of 'local incommensurability' attempting to divorce conceptual change from issues of reference (Kitcher, 1983)[3]. She identified several kinds of conceptual changes that could be considered as radical conceptual change and provided the empirical evidence to support the claim that they occur in the course of spontaneous cognitive development. Conceptual change according Carey (1991) requires the re-assignment of a concept to a different ontological category or the creation of new ontological categories – as when the concept of the 'earth' becomes subsumed under the category of astronomical objects as opposed to physical objects (Vosniadou & Skopeliti, 2005)—it can also involve the differentiation or coalescence of concepts – such as the differentiation between heat and temperature or weight and density (Carey, 1991; Carey & Spelke, 1994; Wiser & Carey, 1983;Wiser & Smith, this volume).

Other researchers have argued that there are many different kinds of conceptual changes that happen in the process of learning and development as well as in the history of science (Thagard, 1992; this volume). Thagard (this volume) provides such a list of conceptual changes, starting with some of the simpler kinds, i.e., those that involve adding a new instance or a new rule to an existing concept,

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1 For criticisms of Kunh's views in the philosophy and history of science, see Arabatzis and Kindi (this volume) and Machamer (2007). A more detailed description of our views on conceptual change and incommensurability will be provided later in this chapter.

2 Similar views were simultaneously and independently expressed by the philosopher Paul Feyerabend (1962).

3 Global incommensurability can easily lead to an anti-realist position. If concepts change then it is not clear how they can continue to refer to the same entities or processes. Adopting a realist stance requires an account of concepts that keeps reference constant as the meaning of the concepts changes (see also Putnam, 1992).

and ending with some of the more radical. The latter are the ones requiring changes in the ontological category tree in which a concept belongs, with corresponding changes in causality – as, for example, when concepts like ‘life’, ‘disease’ and ‘mind’ change from being embedded within theological explanations to qualitative and then to mechanistic frameworks. Chi (this volume) also distinguishes between three kinds of conceptual changes that happen in the process of learning: belief revision, mental model transformation, and categorical shift. Similar distinctions are also made by Keil and Newman (this volume), Wiser and Smith (this volume), Inagaki and Hatano (2003; this volume), to characterize the process of cognitive development.

The focus of the present chapter is to describe and explain the kinds of conceptual changes that take place when students are exposed to counter-intuitive concepts in science and mathematics (Vosniadou, 2006; Vosniadou & Verschaffel, 2004). We are interested in what Inagaki and Hatano (this volume) call ‘instruction induced conceptual change’, as compared to the kinds of conceptual changes that happen spontaneously in development. We argue that many science concepts are difficult to learn because they are embedded within scientific theories that violate fundamental principles of the naïve, framework theory of physics within which everyday physics concepts are subsumed. In other words, the learning of many science concepts requires the more radical kind of conceptual changes that involve ontological category shifts.

At the heart of our theoretical approach is the idea that initial explanations of the physical world in naive physics are not fragmented observations but form a coherent whole, a *framework theory*. The change of the framework theory is difficult because it forms a coherent explanatory system, it is based on everyday experience, and it is constantly re-confirmed by our everyday experiences in the context of lay culture. After all, the currently accepted scientific explanations are the product of a long historical development of science characterized by radical theory changes that have restructured our representations of the physical world. More recently we have started to work in the area of mathematics. Although the domain of mathematics is very different from that of science, we believe that the same analysis can roughly apply in the case of learning mathematics (see Vosniadou & Verschaffel, 2004; Verschaffel & Vosniadou, 2004)

The first section of this paper presents two examples of conceptual change, one coming from the area of physics (observational astronomy) and the other from the area of mathematics (rational number). The second section provides a more detailed analysis of the theoretical position which also explains its similarities and differences with other approaches dealing with the problem of conceptual change in learning and development. The paper concludes with a discussion of some of the implications of the framework theory approach for the design of instruction in science and mathematics.

It is our contention that the problem of conceptual change commonly posed in instruction is one of the major reasons behind students' widespread failure to understand concepts in science and mathematics. An overwhelming body of educational research has documented the considerable difficulties students encounter in these areas. These difficulties are not present only in the case of the weaker or younger students. They are present even in the brighter college students, attending the most prestigious universities. Absence of critical thinking, knowledge fragmentation, lack of transfer, and misconceptions characterize the reasoning and problem solving of many students, particularly in those cases where the new, to-be-acquired information conflicts with the structure of existing, experience-based, lay knowledge. Finally, it is also our contention that, to a large extent, the general ineffectiveness of instructional interventions in this area could be attributed to the inadequate attention that has been given so far to the problem of conceptual change.

Two Examples of Instruction-Induced Conceptual Change

The concept of the earth[4]

Children's initial concept

A substantial body of cross-cultural research supports the conclusion that during the preschool years children construct an initial concept of the earth based on interpretations of everyday experience in the context of lay culture. According to this initial concept, the earth is a flat, stable, stationary, and supported physical object. Objects located on the earth obey the laws of an up/down gravity, and space is organised in terms of the dimensions of up and down. The sky and solar objects are located above the top of this flat earth which is thought to occupy a geocentric universe (see Brewer, this volume; Vosniadou & Brewer, 1992; 1994; Nussbaum, 1979; 1985).

Scientific concept

[4] The term 'concept' is used here to denote both individuals' concepts and the socially shared and culturally accepted scientific concepts.

As shown in Table 1, the scientific concept of the earth, to which children are exposed at least as soon as they enter elementary school, violates practically all of the presuppositions that apply to children's initial earth concept. According to the scientific concept, the earth is a planet – an unsupported, spherical, astronomical object – which rotates around its axis and revolves around the sun in a heliocentric solar system. People live all around the spherical earth and gravity operates towards the center of the earth. Understanding the scientific concept of the earth requires that children re-categorize the earth to a new ontological category -- from a physical object to an astronomical object. We consider this re-categorization to be a form of radical conceptual change.

"Insert Table 1 about here"

Conceptual change

The hypothesis that the acquisition of the scientific concept of the earth requires conceptual change was tested directly in an empirical study by Vosniadou & Skopeliti (2005). In this study 62 1st and 5th grade children were shown 10 cards with the words 'sun, moon, star, earth, planet, house, cat, rock, tree, and car' and were asked three categorization questions. The results, which are described in detail in Table 2, showed that great majority of the children were able to distinguish physical from solar objects and that there was a developmental shift in their categorizations of the earth.

"Insert Table 2 about here"

Children's responses, particularly in the third question, which asked them explicitly to put together the things that go with the earth in one group and the things that do not go with the earth in a different group, were very revealing. At grade 1, 35% of the children categorized the earth as a physical object and 42% as a solar object, while at grade 5 only 1 child categorized the earth as a physical object and 90% categorized it as a solar object. Further, children's responses to an earth shape questionnaire similar to the one used by Vosniadou & Brewer (1992) showed significant correlations between children's categorizations and their earth shape models.

We concluded that the results support the hypothesis that there is a change in the categorization of the earth from a physical to a solar object, and that the re-categorization of the earth as a solar object may precede children's full understanding of the earth as a spherical planet, rotating around its axis and revolving around the sun.

Internal Inconsistency and Synthetic models

The re-categorization of the earth as a solar object does not take place overnight. A series of cross-sectional developmental studies (e.g., Blown & Bryce, 2006; Diakidoy, Vosniadou & Hawks, 1997; Nussbaum, 1979; Nussbaum & Novak, 1976; Vosniadou & Brewer, 1992, 1994) as well as a some longitudinal studies (Kikas, 1998; Maria, 1993; 1997a, 1997b), support the hypothesis that the process of acquiring the scientific concept of the earth is a slow and gradual process which gives rise to the construction of alternative conceptions of the earth as well as to internally inconsistent responses. A list of the alternative representations constructed by elementary school children in the Vosniadou and Brewer (1992) study appears in Figure 1. These accounted for about 90% of the overall responses of the 3rd and 5th grade children and 65% of the overall responses of the 1st grade children. The overall responses of the remaining children were categorized as mixed[5].

"Insert Figure 1 about here"

The internally consistent responses formed a range of alternative models of the earth, starting from the initial representation of a flat earth to the scientific representation of a sphere. The younger children tended to represent the earth as a square, rectangle, or disc-like flat, physical object, supported by ground below and the sky and solar objects above its top. Some children formed the interesting model of a dual earth, according to which there are two earths: a flat one on which people live and a spherical one which is up in the sky and which is a planet. Another common mis-representation of the earth was that of a hollow sphere. According to that model, the earth is spherical but hollow inside. People live on flat ground inside the bottom part of the hollow sphere. Alternatively, the earth was conceptualized like a flattened sphere or truncated sphere with people living on its flat top, coved by the dome of the sky above its top.

These alternative representations of the earth were not rare. In fact, only 23 of the 60 children in the Vosniadou & Brewer (1992) study had constructed the culturally accepted representation of the earth as a sphere. These findings have been confirmed by many cross-cultural studies conducted both in our lab (e.g., Diakidoy, Vosniadou & Hawks, 1997; Samarapungavan, Vosniadou & Brewer, 1996; Vosniadou, Skopeliti & Ikospentaki, 2004; 2005), as well as by a number of independent investigators (Blown & Bruce, 2006; Hayes, Goohew, Heit, & Gillan, 2003; Mali & Howe, 1979)[6].

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5 The children placed in the 'mixed' category usually gave a mixture of 'scientific' and 'naïve' responses, just like the children grouped under the 'alternative models' category, but their responses were non-systematic and often self-contradictory.

6 There are a number of studies by Siegal, Nobes and their colleagues (e.g., Nobes, Martin & Panagiotaki, 2005; Siegal, Butterworth, Newcombe, 2004) criticizing the above-mentioned findings. For extensive and detailed discussions of the theoretical and methodological issues around these studies raise please see Vosniadou, Skopeliti and Ikospentaki (2004, 2005) as well as Brewer (this volume).

The process of conceptual change

Given that the spherical representation of the earth is so ubiquitous in our culture, one wonders why children have such great difficulty understanding it and why they form the alternative representations noted above. The explanation we have given is that the change from a flat earth to a spherical earth concept is not a change in a simple belief, but a radical conceptual change. This is because the initial concept of the earth is embedded within a larger, framework theory of physics, forming a complex construction which is supported by a whole system of observations, beliefs and presuppositions constituting a relative coherent and systematic explanatory structure (Vosniadou & Brewer, 1992; 1994; Vosniadou, 2007b). Figure 2 shows some of the observations, beliefs and presuppositions of the assumed conceptual structure that underlies the initial concept of the earth.

"Insert Figure 2 about here"

Two assumptions are made here which will be discussed in greater detail in the next section. One is that the concept of the earth is embedded within a domain-specific, framework theory of physics, i.e., a naïve physics. The second is that categorization is a powerful process that plays an important role as a mechanism in learning (Bransford, Brown, & Cocking, 1999; Chi, this volume; Chi & Koeske, 1983; Medin & Rips, 2005). Knowing that an object belongs to a given category allows us to infer certain characteristics of the object which can either support learning or hinder it, if the category to which it is assigned is inappropriate. In the case of the earth, its categorization as a physical object allows young children to make a host of inferences about the way they interpret observational evidence received from experience and draw conclusions regarding certain inaccessible, unobservable properties of the earth (e.g., that it is supported or that it has an end). These inferences are not subject to conscious awareness and can stand as powerful presuppositions constraining the process of learning science.

An examination of children's alternative models of the earth, as well as their internally inconsistent responses, suggests that children use enrichment type of learning mechanisms to add the new (scientific) information to their initial concept of the earth. While the use of such mechanisms can be very appropriate in most situations where the new, to-be-acquired, information is consistent with what is already known, they are not very productive when the new information belongs to a scientific concept embedded within a theoretical framework incompatible – incommensurate we might say – with children's initial concept of the earth.

As we have argued so far, the scientific concept of the earth is embedded within a different explanatory framework – that of an astronomical object—a framework that differs in many of its presuppositions from the presuppositions of the initial concept of the earth, which is categorized as a physical object embedded within a naïve physics. In cases such as these, where the new information comes in conflict with what is already known, the use of additive, bottom-up enrichment mechanisms can only lead the learner into small changes which may either fragment what is already known creating internally inconsistent pieces of knowledge, or at best lead into the creation of alternative models or misconceptions.

We have interpreted the alternative models of the earth to be 'synthetic models' because they seem to result from children's attempts to synthesize the information that comes from the scientific concept, and particularly the information that the earth is a sphere, with aspects of the initial concept of the earth, i.e., that it is a solid, stable, supported physical object, with an up/down organization of space and gravity. If we look carefully at all the alternative models of the earth in figure 1, we can see that in all cases they represent attempts on the part of the children to solve the problem of how it is possible for the earth to be spherical and flat at the same time and how it is possible for people to live on this spherical earth without falling down.

Furthermore, the process of conceptual change appears to involve a gradual lifting of the presuppositions of the framework theory allowing the formation of more sophisticated models of the earth, until conceptual change has been achieved. Although most of the empirical support for synthetic models comes from cross-sectional studies, the developmental pattern is clear (see also Brewer, this volume). The less sophisticated and more fragmented responses occur at the younger ages while the more sophisticated synthetic models, and of course the scientific model, are found in the older children. Thus, children start with the model of a square or rectangular, supported, stable, and flat earth that meet all the presuppositions of the earth as a physical object. The model of the disc earth shows some possible influence from the culture reflected in the change in the assumed shape of the earth from rectangular to round (but flat). This model could be an initial model or it could result from some exposure to scientific information about the shape of the earth. The dual model of the earth is an interesting construction that shows how scientific information can be incorporated into the knowledge base in a way that does not affect existing knowledge structures that contradict it. In this model the children believe that the spherical earth is different from the flat earth on which we live – it is a planet up in the sky.

The models of the hollow sphere and the truncated sphere are more sophisticated and are formed usually by older children. The model of the hollow sphere presupposes an understanding that the earth is spherical and not supported, but it is constrained by an up/down gravity presupposition. The children who construct this model believe that people live on flat ground inside the earth because they would fell "down" if they lived on the surface of the spherical earth. Similarly, the up/down gravity presupposition constrains the understanding of the spherical earth in children who have constructed a flattened or truncated sphere, who also believe that people live on flat ground above the top of the earth. These children can see the earth as a spherical, suspended and sometimes rotating object but they still organise space and gravity in terms of the directions of up/down. These are some of the most fundamental presuppositions of a naïve, framework theory of physics.

It should be mentioned here that the children placed in the category 'mixed' also use enrichment types of learning mechanisms to add the new information to their initial concept. The difference from the 'synthetic' models category is that the children placed in the mixed category were either not aware of the internal contradiction in their responses or could not find a way to solve this contradiction through the creation of a 'synthetic model'. Synthetic models are actually quite creative constructions as they provide unique solutions to the problem of incommensurability and have explanatory power. In order to avoid internal inconsistency or the creation of synthetic models the learner must first of all become aware of the incongruity that exists between the incoming information and his/her prior knowledge. Metaconceptual awareness and intentional learning are required for conceptual change to be achieved. Learners must also avoid the use of simple, additive mechanisms. Conscious, intentional and top-down learning mechanisms, such as the deliberate use of analogy and cross-domain mappings, are much better mechanisms for producing radical conceptual change. These issues will be discussed in greater detail in the last section of the paper.

The concept of number

Children's initial concept

It appears that children form an initial concept of number which allows them to deal with number-related tasks long before they are exposed to formal instruction in mathematics. Summarizing the relevant empirical findings, Gelman (1994) concludes that by four or five years of age children are able to "count in principled ways, invent solutions to novel counting problems, detect errors in counting trails generated by others and make up counting algorithms to solve simple addition and subtraction problems, at least for a limited range of nthe numbers" (p.68). These abilities reflect a concept of number close to the mathematical concept of natural numbers.

The first years of instruction are dedicated to natural number arithmetic. Thus, the initial understandings of number are compatible with school instruction. As a result, the natural number concept is further confirmed and strengthened. By the middle of the elementary school years most children have built a rich and productive number concept, which is based on counting, and which carries all the basic presuppositions of natural numbers described in Table 3.

A basic characteristic of this initial understanding of numbers is that numbers are discrete i.e., that every number has a unique successor. In fact, there seems to be some evidence that the property of discreteness of numbers may be neurobiologically based, in the sense that humans are predisposed to learn and reason with natural numbers (Dehaene, 1998; Gelman, 2000). Another characteristic is that numbers can be ordered by means of their position on the count list, with numerals with more digits corresponding to bigger numbers (see also Smith, Solomon, & Carey, 2005). Numbers are involved in the operations of addition and subtraction which can be supported by counting-based strategies (e.g. Resnick, 1986). The operation of multiplication is interpreted as repeated addition while division is interpreted as partitioning, where the divisor is smaller than the dividend (Fischbein et al., 1985). All four operations are seen as having predictable outcomes, in the sense that addition and multiplication 'make bigger', whereas subtraction and division 'make smaller' (Fischbein et al., 1985; Moskal and Magone, 2000). Finally, it is also assumed that every number has only one symbolic representation, that there is a unique numeral that corresponds to each number.

"Insert Table 3 about here"

The mathematical concept of rational number

All of the above-mentioned assumptions underlying students' number concept come in contrast with the mathematical concept of rational number introduced through instruction: Rational numbers are not based on counting and they are dense and not discrete. In other words, no rational number has a successor within the rational number set, and there are infinitely many numbers between any two, non equal, rational numbers. Counting-based strategies cannot support the ordering of rational numbers (for example, 1/4 is bigger than 1/5). In addition, 'longer' rational numbers are not necessarily 'bigger' (for instance, 3.2 is bigger than 3.197). Operations with rational numbers do not have predictable results – in the sense that addition and multiplication may result either in bigger, or in smaller outcomes, as in the case of 4+ (-2), or 3x1/2. Multiplication cannot always be conceptualized as repeated addition, (consider the case 0.3x1/2) and it is difficult to understand division as partitioning when the divisor is bigger than the dividend (e.g. 2:8), or when the divisor is smaller than one (e.g. 2:0.5).

Finally, rational numbers do not have only one symbolic representation. Rather, they can be represented symbolically either as decimals, or as fractions. For example, the number "one half" can be represented as 0.5 and also as 1/2. To make things more complicated, the one half can be represented also as 0.50, 0.500, 2/4, 4/8, etc. This presents the learner with yet another difficulty: one must realise that fractions and decimals[7] are alternative representations of rational numbers (and not different kinds of numbers), despite their differences in notation, ordering, operations and contexts of use.

Conceptual change

While there is no direct evidence of theory change that we know of, there is a great deal of empirical evidence showing that rational number reasoning is very difficult for students at all levels of instruction and in particular when new information about rational numbers comes in contrast with prior natural number knowledge (Moss, 2005; Ni & Zhou, 2005). For instance, many students believe that 'longer decimals are bigger' (e.g. Moskal and Magone, 2000), that 'multiplication makes bigger' and 'division makes smaller' (Fischbein et al., 1985), or that 'the bigger the terms of a fraction, the bigger the fraction' (e.g. Stafylifou & Vosniadou, 2004). Students at elementary, secondary and even university levels do not realize that rational and real numbers are dense (e.g. Malara, 2001; Merenluoto & Lehtinen, 2002, 2004; Neumann, 1998; Tirosh, Fischbein, Graeber, & Wilson, 1999; Vamvakoussi and Vosniadou 2004, 2007, in preparation). They have many difficulties interpreting and dealing with rational number notation, in particular when it comes to fractions (Gelman, 1991; Moss, 2005;

[7] The term 'decimal' is used here to refer only to the decimal numbers that belong to the rational numbers set.

Stafylifou & Vosniadou, 2004). They do not realize that it is possible for different symbols (e.g., decimals and fractions) to represent the same number and thus they treat different symbolic representations as if they were different numbers (Khoury & Zazkis,1994; O'Connor, 2001; Vamvakoussi & Vosniadou, 2007).

Internal inconsistency and synthetic models

Some of the difficulties secondary school students have in understanding rational numbers were investigated in a series of studies in our lab, focusing mostly on the discreteness/density divide and its interaction with students' interpretation of rational number notation (Vamvakoussi & Vosniadou 2004, 2007, in preparation). It was hypothesized that the presupposition of discreteness which is a characteristic of the initial concept of number, would constrain students' understanding of rational numbers causing fragmentation, internal inconsistency and misconceptions which could be interpreted as 'synthetic models'. This hypothesis is consistent with the existing empirical evidence for students (e.g., Malara, 2001; Merenluoto & Lehtinen, 2002, 2004; Neumann, 1998), as well as for prospective teachers (e.g. Tirosh, Fischbein, Graeber, & Wilson, 1999). This is because the initial concept of number forms a coherent explanatory system that produces correct predictions and explanations in most everyday situations where number reasoning is required.

It was also predicted that students would have difficulty understanding that decimals and fractions are interchangeable representations of rational numbers, and not different kinds of numbers. This is the case because a) decimals and fractions take their meaning from the situations to which they refer and these situations are usually qualitatively different (Resnick, 1986), b) there are considerable differences both between the operations and between the ordering of decimals and fractions, and c) students may categorize rational numbers on the basis of their notation, which may be considered a superficial characteristic by the mathematically versed person, but not by the novices in the domain (see Markovitz and Sowder, 1991, Chi, Feltovich, & Glaser, 1981).

The results confirmed that the presupposition of discreteness is strong for the younger students (7th graders) and remains robust even for older students (9th and 11th graders), despite noticeable developmental differences. Students from all age groups answer frequently that there is a finite number of numbers in a given interval, regardless of whether they are asked in an interview (Vamvakoussi & Vosniadou, 2004), in an open-ended questionnaire (Vamvakoussi & Vosniadou,

2007) or a forced choice questionnaire (Vamvakoussi & Vosniadou, 2007, in preparation). Figure 3 presents the distribution of the 549 participants in our third study (Vamvakoussi & Vosniadou, in preparation), based on the number of 'finite' versus 'infinite' responses they gave to the question "How many numbers are there between X and Y", where the pair 'X and Y' could be integers, decimals or fractions (in a total of 10 items, consisting of 2 integer, 4 decimal and 4 fraction questions). FIN students gave at least 7 'finite' responses while INF students gave at least 7 'infinite' responses. FIN/INF included all the remaining students. As can be seen, the 7th grade students gave mostly finite responses, while the older students were placed mostly in the FIN/INF and FIN categories. Nevertheless 30% of the 9th and 11th grade students were still categorised in the most naïve, FIN category.

"Insert Figure 3 about here"

Second, the results showed that the presupposition of discreteness is not 'lifted' overnight. In other words, it is not the case that students become aware of the infinity of numbers at some point and then consistently apply it to all the given intervals. Rather, there seems to be a pattern of development during which information regarding the dense structure of numbers is slowly added on to existing conceptual structures: This developmental pattern seems to be roughly the following: a) the presupposition of discreteness is lifted first for integers and then for decimals and fractions, b) there is some indication that students apply the notion of infinity first to decimals and then to fractions, and c) infinity seems to be initially restricted to numbers of the same symbolic representation within an interval, i.e., only decimals between decimals and integers, and only fractions between fractions.

To illustrate these points, a more refined sub-categorization of the participants within the categories FIN, FIN/INF and INF in the Vamvakoussi and Vosniadou (in preparation) study is presented in Table 4. As can be seen, we can identify an *initial, all finite model* in the category FIN, as well a kind of an *integers only* synthetic model consisting of students who seem to apply the notion of infinity in integers but not in decimals and fractions. In the category FIN/INF we see the emergence of the *decimal –fraction distinction.* Students in this sub-category answer differently when the interval ends are fractions, as compared to when they are decimals. More specifically, these students may a) give an INF answer for decimals, but a FIN answer for fractions, or b) give an INF answer for decimals, but an FIN answer for fractions. Usually, this distinction is in favour of decimals. Finally in the INF category the most interesting distinction is between the *advanced model* and the *sophisticated* *model.* In both of these models the students give only INF responses, however, in the *advanced model* they prefer to place decimals between decimals and fractions between fractions, while in the *sophisticated model* they are willing to accept that they may have different symbolic representations.

"Insert Table 4 about here"

To sum up, it appears that students form an initial understanding of number roughly as natural number. This initial concept then stands in the way of understanding the concept of rational number presented to students through instruction. Understanding the mathematical concept of rational number requires a re-structuring of the initial concept of number. This is not easy to happen. Students need to realize that certain presuppositions, like the discreteness of numbers, are valid only in specific contexts. Also, learning about rational numbers requires from students to construct meanings for new symbolic notations -- fractions and decimals – not encountered before (Gelman, 1991; Stafylidou & Vosniadou, 2004). In addition to the difficulty of interpreting decimal and fractional notation in its own right, students have to realize that different symbolic notations refer to the same object, i.e., that decimals and fractions are interchangeable representations of rational numbers, and not different kinds of numbers.

The empirical evidence suggests that students are using conservative, additive, enrichment types of mechanisms to add new information to existing but incompatible conceptual structures. These mechanisms create internal inconsistency and 'misconceptions' of the rational number concept which can be explained as 'synthetic models'. One of these synthetic models is to conceptualize the rational numbers set as consisting of three different and unrelated 'sets' of numbers: whole numbers, decimals, and fractions. As they come to understand the principle of density through instruction, students then apply this principle additively to the different 'sets' of numbers. As a result, students come to think differently for integers, decimals, and fractions in terms of their structure (discrete vs. dense). They also become reluctant to accept that there might be fractions between two decimals, or vice versa. This phenomenon has been noted also by Khoury and Zazkis (1994) and reported by O'Connor (2001) as a fact noticed by mathematics teachers.

Summary

Despite the enormous differences in the two concepts we analyzed, which are embedded in very different domains of thought, we can observe certain important similarities. In both cases we have a situation where the new concept, regardless of whether it is of a scientific or a mathematical nature, comes in conflict in practically all its major ontological presuppositions with the expected prior knowledge of the student. In both cases there is adequate empirical evidence to support the conclusion that an initial concept of the earth and an initial concept of number is constructed early on, in the process of spontaneous knowledge acquisition on the basis of everyday experience in the context of lay culture. This concept is embedded within a naïve theory of physics or of number that forms a narrow but relatively coherent explanatory structure. The new information presented through instruction comes in conflict with the presuppositions of the existing framework theory. Being largely unaware of this conflict students assimilate the new information into the existing but incompatible knowledge base using enrichment type mechanisms which result in internally inconsistent responses or in the formation of synthetic models. Enrichment type mechanisms can be very successful in many cases of knowledge acquisition but fail in situations that require conceptual change, because of the incompatibility between the way knowledge is structured in the students' knowledge base and the structure of the scientific or mathematical concept presented through instruction.

The framework theory approach

The framework theory approach is based on cognitive/developmental research and attempts to provide a broad theoretical basis for understanding how conceptual change is achieved in the process of learning science. There are certain fundamental assumptions which characterize this approach which are discussed below. In short, it is claimed that there is enough empirical evidence coming from research in cognitive development to support they view that concepts are embedded in domain specific 'framework theories' which represent different explanatory frameworks from currently accepted science and mathematics (see Carey, 1991; Carey & Spelke, 1994; Hatano, 1994; Keil, 1994). These framework theories are constructed early on and are based on children's interpretations of their common everyday experiences in the context of lay culture. Because learners use additive, enrichment types of learning mechanisms to assimilate the new incompatible information to existent knowledge structures, the process of learning science and mathematics is slow and gradual and characterized by fragmentation, internal inconsistency and misconceptions, some of which can be interpreted as 'synthetic models' (Vosniadou & Brewer, 1992; 1994; Vosniadou, Baltas & Vamvakoussi, 2007).

Domain specificity

Most theories of learning and development, such as Piagetian and Vygotskian approaches, information processing or socio-cultural theories are *domain general*. They focus on principles, stages, mechanisms, strategies, etc., that are meant to characterize all aspects of development and learning. In contrast, domain specific approaches focus on the description and explanation of the changes that take place in the content and structure of knowledge with learning and development as well as on mechanisms and strategies that are specific to these changes.

The idea that human cognition includes domain-specific mechanisms for learning is based on a number of independent research traditions and sets of empirical findings, some coming from animal studies (Gallistel, 1990), others based on Chomsky's work in linguistics (Chomsky, 1988). Some cognitive developmental psychologists see domain specificity through the notion of domain specific *constraints* on learning (Keil, 1981, 1990). It is argued that such constraints are needed in order to restrict the indeterminacy of experience (Goodman, 1972) and guide, amongst others, the development of language (Markman, 1989), numeric understanding (Gelman, 1990), or physical and psychological knowledge (Wellman & Gelman, 1998).

There is a great deal of debate in the literature as to whether domain specific constraints should be seen as hardwired and innate as opposed to acquired, and as having representational content or not (see Elman et al., 1996). Hatano and Inagaki (2000) suggested that constraints are innate domain specific biases or preferences that mitigate the interaction between a learning system and the environment. They also introduced the notion of 'socio-cultural constraints'. They argue that socio-cultural factors can also guide learning and development by restricting the possible range of alternative actions thus leading the learner to select the most appropriate behaviour (see also Hatano & Miyake, 1991; Keil, 1994).

Finally, some domain specific approaches focus on the description of the development of expertise in different subject-matter areas, such as physics (Chi, Feltovitch, & Glaser, 1981), mathematics (VanLehn, 1990; Mayer 1985) or chess (Chase & Simon, 1973), without necessarily appealing to innate modules or constraints.

Our position is that it is more profitable to study learning from a domain specific point of view that allows us to make hypothesis about the way a specific content is structured (and re-structured), without necessarily committing ourselves to innate constraints or modules. We also believe that domain-specific approaches should be seen as complimentary rather than contradictory to domain general approaches. It is very likely that both domain general and domain specific mechanisms apply in development and learning (Keil, 1994).

Framework theories, specific theories and mental models

The human child is a complex organism capable of engaging in quick and efficient learning immediately after birth. Cognitive developmental research has provided substantial empirical evidence to support the view that children organize the multiplicity of their sensory experiences under the influence of everyday culture and language into narrow but relatively coherent, domains of thought from very early on (Baillargeon, 1995; Carey & Spelke, 1994; Gelman, 1990). It appears that at least four well defined domains of thought can be distinguished and considered roughly as 'framework theories' – physics, psychology, mathematics, and language.

Each one of these domains has its unique ontology -- applies to a distinct set of entities. For example, physics applies to inanimate bodies, psychology applies to animate entities, mathematics to numbers and their operations and language to lexical items and their operations. Each domain is also governed by a distinct system of principles and rules of operation. Physical entities obey the laws of mechanical causality as opposed to psychological entities that are governed under intentional causality. Language and mathematics have their own unique rules and principles of organisation. We are not going to delve further into these differences here[8]. The important point to make is that in all these cases we are not dealing with a collection of unrelated pieces of knowledge but with coherent and principle-based systems.

Each one of these domains of thought has certain procedures for identifying the entities that belong to the domain. For example, it appears that the criterion of self-initiated vs. non-self-initiated movement is used by infants to distinguish physical from psychological entities. Once categorized as a physical or psychological object, an entity inherits all the characteristics and properties of the entities that belong to the domain. As mentioned earlier, categorization is a very important learning mechanism in this respect. We assume that concepts are embedded in framework theories (such as a naïve physics, psychology, mathematics, etc.) and that they inherit all the properties of the framework theory to which they belong. In addition, they may contain additional information which belongs specifically to the concept – has the form of a 'specific theory'. The hypothesized internal structure of

[8] Language development does not seem to be characterised by theory-changes, but some of the difficulties students experience in learning a foreign (or second) language may be of a similar nature.

the initial concept of the 'earth' is described in figure 2. This structure includes specific information about the earth, coming from observation and from the culture (i.e., that the earth is flat, the sky is above the earth, etc.), but interpreted within the constraints of the framework theory. The concept of the earth is not stable (although reference is), but evolves and develops with knowledge acquisition, with changes happening both at the level of the specific theory and at the level of the framework theory.

Finally, we assume that human beings have a cognitive system that allows them to create analog mental representations of physical objects that embody the internal structure of the concept and can be run in the mind's eye to generate predictions and explanations of phenomena (see Nerserssian, this volume for a discussion of some of these issues). For example, we can create a mental model of the earth and we can use this model to answer questions like 'What will happen if you were to walk for many days on the earth? Is there an end/edge to the earth? Can you fall from this earth?' Depending on our mental model of the earth we can answer this question in different ways. In our previous work (Vosniadou & Brewer, 1992; 1994 – see also Brewer, this volume) we provided numerous examples that demonstrate beyond any possible doubt that even very young children are capable of using the earth, the moon and the sun as theoretical entities in models that can be run in the mind to make predictions and provide explanations of phenomena.

Conceptual change

There is substantial evidence that cognitive development is characterized by conceptual change. For example, in the domain of biology, cross sectional developmental studies show that the biological knowledge of the 10 year old is qualitatively different from that of the 4-6 year old child (Carey, 1985; Carey, 1991; Keil, 1994; Hatano and Inagaki, 1997; Inagaki & Hatano, 2003; this volume), although there is considerable disagreement as to how exactly this development proceeds. Theory changes in the domain of biology have been described in terms of three fundamental components: the ontological distinctions between living/non-living and mind/body, the modes of inference that children employ to produce predictions regarding the behaviour of biological kinds, and third, the causal-explanatory framework children employ -- e.g., intentional as opposed to vitalistic or mechanistic causality (Inagaki & Hatano, 2003).

Similar re-organizations of conceptual knowledge across early childhood years can be found amongst others, in children's concept of mind (Wellman, 1990), concept of matter (Smith, Carey, & Wiser, 1985; Wiser & Smith, this volume), concept of force (Chi, 1992; Ioannides & Vosniadou, 2002), concept of number (Smith, Solomon & Carey, 2005), and concept of the earth (Vosniadou & Brewer, 1992, 1994). As described earlier, the empirical evidence in the area of observational astronomy has shown that considerable qualitative changes take place in children's concept of the earth between the ages of 4-12. Pre-school children think about the earth as a stable, stationary and flat physical object located in the centre of the universe. On the contrary, most children at the end of the elementary school years think of the earth as a spherical astronomical object, rotating around it and revolving around the sun in a heliocentric solar system. In this process, a significant ontological shift takes place in the concept of the earth which is categorized as a *physical object* by the majority of first graders but as a *solar, astronomical object* by the majority of sixth graders (Vosniadou & Skopeliti, 2005). Similar ontological shifts have been pointed out in the case of the concept of force and of heat amongst others (Chi, 1992, Ioannides & Vosniadou, 2002; Wiser & Carey, 1983).

Mechanisms of conceptual change and the problem of incommensurability

Conceptual change can happen either through the use of bottom-up, implicit and additive mechanisms, or through the use of top-down, deliberate and intentional learning mechanisms, assuming of course, a continuous interaction between an individual and a larger, surrounding cultural context. Examples of the former can be mechanisms like the Piagetian assimilation and accommodation, the use of similarity-based analogical reasoning (Vosniadou, 1989), internalization (Vygotsky, 1978), or even the appropriation of cultural practices of the situated theorists (Rogoff, 1990). Examples of the latter are the deliberate use of analogy and models that allow mappings across domains, the construction of thought experiments and limited case analyses, and translations from physical language to the language of mathematics, (see Carey & Spelke, 1994; Nersessian, 1992; Vosniadou, 2007c). Very important are also to mention several social kinds of mechanisms that can facilitate conceptual change, like collaboration (Miyake, this volume) and class discussion (Hatano & Inagaki, 2003).

According to Carey (1991) ordinary, intuitive, cognitive development involves radical conceptual changes of the nature described earlier which cannot be explained if we assume that children only use enrichment types of mechanisms. She agrees with Spelke (1991) that enrichment types of mechanisms cannot produce radical conceptual change but can only form new beliefs over concepts already available. In Carey and Spelke (1994) the evidence supporting the claim for spontaneous conceptual change is again reviewed and certain possible mechanisms are discussed that are mainly based in mappings across different domains of thought. For example, changing conceptions of number (from natural number to rational number) are thought to depend on the construction of mappings between number and physical objects as the child learns measurement (see also Gelman, 1991), and that the development of mechanistic biology and mechanistic psychology require mappings from the domain of psychology to physics.

We also believe that cross domain mappings and the use of thought experiments and limited case analyses are powerful mechanisms for conceptual change that they should be encouraged in instruction. However, the results of many empirical studies show that most of the conceptual changes that happens spontaneously in cognitive development are the product of enrichment types of mechanisms, that are not under the conscious control of the learner. These types of mechanisms are capable of producing radical conceptual change if we assume a) that the knowledge base has a theory-like internal structure, and b) new information is coming in through observation and from the culture. In other words, they presuppose that children will grow in a culture with a developed science and that they will be exposed to the re-structured concepts either through participation in the adult lay culture and language or through systematic science and math instruction.

For example, young children categorize plants usually as non-living things. However, everyday experiences with plants, such as watering plants, seeing them become bigger, or noticing that they can die, in the context of an adult culture and language, can slowly lead the children to understand that plants are similar to animals in certain properties, such as feeding, growing, and dying. These similarities can eventually make children re-categorize plants as living things, rather than as inanimate objects, despite the fact that they lack self-initiated movement (Hatano, 1996; 2002). This category change can be described as branch jumping (Thagard, 1988), or as an ontological category shift (Chi, 1992), and represents a considerable re-organization of the concept of living thing (Carey, 1985), that can be characterized as requiring conceptual change. Similar ontological category changes can be produced when children are given direct feedback on plants' capacity for goal-directed movement (see Opfer & Siegler, 2004). In fact, it appears that learning about teleology is more effective than learning about the need for water and food on the categorization of plants, possibly because the former criterion (goal directed movement) is more critical for categorizing an entity as a living thing than the latter (need for food and water).

Although the use of bottom-up, implicit, additive mechanisms can be useful in producing even radical kinds of conceptual changes under conditions of spontaneous cognitive development, they can also be the source of producing internal inconsistency and synthetic models in many instances where 'instruction-induced conceptual change' is needed. This is the case because the teaching of science usually takes place in a school context where students are required to understand in a short period of time counter-intuitive concepts that took several scientific revolutions to be constructed. Furthermore, it is often the case that inappropriate curricula are used by teachers who are not always knowledgeable about the problem of conceptual change and who may not fully understand the magnitude of the difficulties experienced by students (see Duit et al., this volume). In these situations, instruction-induced conceptual change becomes a slow process during which the new, counter-intuitive, scientific, information is assimilated into students' initial concepts, creating internal inconsistency and misconceptions. Many of these misconceptions are *synthetic models*, formed as learners assimilate the scientific information to an existing but incompatible knowledge base without metaconceptual awareness (Vosniadou, 2003, 2007b).

A number of experimental studies in our lab have confirmed that the above mentioned processes are taking place in the learning of science. For example, figure 4 shows the synthetic models of the layers and composition of the earth's interior revealed in the drawings and verbal explanations of students in grades 1, 6, and 11 (Ioannidou & Vosniadou, 2001). Most first graders believe that the earth contains only solid materials (i.e., ground and rocks) arranged in flat layers. Notice that the flat layering representation of the assumed earth's interior is used in children's drawings even in those cases where the earth is seen as round. When the students are instructed about the existence of magma inside the earth, they seem to think that the magma is placed at the bottom, rather than in the center, of the spherical earth. It is only later on that the circular layering appears in their drawings, with the magma placed in the center of the spherical earth. Even the 11th grade students (as well as most undergraduate perspective teachers) believe that magma is located very deep in the center of the earth, rather than relatively close to its surface, and have difficulties understanding the scientific explanations of volcanoes and earthquakes.

"Insert Figure 4 about here"

In another study (Kyrkos & Vosniadou, 1997), we investigated the development of the scientific concept of photosynthesis, which has shown to be a very difficult for students to understand (Barker & Carr, 1989; Haslam & Treagust, 1987;Wandersee, 1983). From the perspective of the framework theory approach, students' difficulties arise from the incommensurability between the scientific and naïve explanatory frameworks of plant development.

As it is shown in Table 5, most first graders consider plants and in the context of a psychological framework theory, explaining plant development through an analogy to animals. More specifically, they think that plants take their food (i.e., water and other nutrients) from the ground through their roots and that they grow as food accumulates in small pieces inside them. As instruction about photosynthesis comes in, this initial explanation becomes fragmented, and a number of different synthetic models can be formed. Some of them are shown in Table 6. One synthetic model is analogous to the 'dual earth' model of the earth. In other words, students retain their initial explanation of how plants grow through feeding, and add to it some information about photosynthesis as referring to a different plant function, that of 'breathing': Plants take in dirty air, they clean it and give out clean air. Another synthetic model is to add to the initial explanation of feeding, a naïve interpretation of photosynthesis. According to this model, plants take food and water from the ground through their roots, but also take food from the air and light through their leaves. A more advanced model of photosynthesis develops in older children who understand that plants make food by themselves but still think of it in terms of mixing elements and not as a chemical process.

"Insert Tables 6 and 7 about here)"

Similarities and Differences with Other Approaches

Piaget's 'global restructuring'

Most approaches to learning, including behaviorist, Piagetian, and Vygotskian sociocultural approaches are domain general approaches based on enrichment type of learning mechanisms. Piaget (1970) has described cognitive development as proceeding through a series of stages, each of which is characterized by a different logical-psychological structure. In infancy, intellectual structures take the form of concrete action schemas. During the preschool years, these structures acquire representational status and later develop into concrete operations --described in terms of groupings based on the mathematical notion of sets and their combinations. The last stage of intellectual development, formal operational thought, is characterized by the ability to engage in propositional reasoning, to entertain and systematically evaluate hypotheses, etc. This type of restructuring applies to all domains of thought and has been characterized as ‘global restructuring’.

Cognitive development according to Piaget is the product of the natural, spontaneous process of constructive intellectual development and not of explicit learning. Nevertheless, experience will be interpreted differently at different stages depending, on the logic of the underlying conceptual structures. The understanding of science concepts is usually thought to require formal operational thinking.

Piaget was instrumental in introducing individual, psychological constructivism (as opposed to social constructivism) to learning research. The importance of prior knowledge and the mechanisms of assimilation, accommodation and equilibration in the context of constructivism are important contributions of Piagetian theory to learning and instruction. Although we agree with the above-mentioned aspects of Piagetian theory, the conceptual change approach described in this paper, differs in important ways from Piaget’s views. The differences have mainly to do with the emphasis on knowledge acquisition in specific subject-matter areas and the notion of ‘domain-specific’ as opposed to ‘global restructuring’. The present approach focuses on knowledge acquisition in specific subject-matter areas and describes the learning of science concepts as a process that requires the significant re-organization of existing domain-specific knowledge structures. Emphasis is placed on the one hand the influence of initial framework theories on the learning process, and on the other on the importance of social, cultural and educational environments in the re-structuring process.

The ‘classical approach’ to conceptual change

The first attempts to interpret Kuhnian theory in science education resulted in the ‘classical approach’ which claimed that the learning of science involves the replacement of persistent, theory-like misconceptions (McCloskey, 1983a; 1983b; Posner Strike, Hewson and Gertzog, (1982). Misconceptions were defined as student conceptions that produced systematic patterns of error. Misconceptions were seen as being either the result of instruction or as ‘preconceptions’ originating prior to instruction. Posner, et al. (1982) drew an analogy between Piaget's concepts of assimilation and accommodation and the concepts of "normal science" and "scientific revolution" offered by philosophers of science such as Kuhn (1962) and derived from this analogy an instructional theory to promote "accommodation" in students' learning of science. The work of Posner et al. (1982) became the leading paradigm that guided research and practice in science education for many years.

Smith, diSessa, & Rochelle (1993) criticized the misconceptions position on the grounds that it presents a narrow view of learning that focuses only on the mistaken qualities of students' prior knowledge and ignores their productive ideas that can become the basis for achieving a more sophisticated mathematical or scientific understanding. Smith et al. (1993) argued that misconceptions should be reconceived as faulty extensions of productive knowledge, that misconceptions are not always resistant to change, and that instruction that "confronts misconceptions with a view to replacing them is misguided and unlikely to succeed" (p. 153).

We agree with the attempts by diSessa (1993) and Smith et al. (1993) to provide an account of the knowledge acquisition process that captures the continuity one expects with development and has the possibility of locating knowledge elements in novices' prior knowledge that can be used to build more complex knowledge systems. We also agree with their proposal to move from single units of knowledge to systems of knowledge that consist of complex substructures that may change gradually indifferent ways. Finally, we agree with Smith et al's (1993) urge to researchers to "move beyond the identification of misconceptions" towards research that focuses on the evolution of expert understandings and particularly on "detailed descriptions of the evolution of knowledge systems over much longer durations than has been typical of recent detailed studies (p. 154).

It could be argued that the 'framework theory' approach we propose is really not very different from the traditional misconceptions position criticised by Smith et al (1993). But this is not the case. Our position meets all the criticisms of Smith et al. (1993). First, we are not describing unitary, faulty conceptions but a complex knowledge system consisting of presuppositions, beliefs, and mental models organised in theory-like structures that provide explanation and prediction. This system is not static but constantly developing and evolving and influenced by students' experience and the information they receive from the culture. Second, we make a distinction between initial concepts, based on initial framework theories, prior to instruction, and those that result after instruction. We argue that information presented through instruction can cause students to become internally inconsistent or to form misconceptions and synthetic models. This is the case because the new information is simply added on to prior but incompatible knowledge through constructive, enrichment type mechanisms. Synthetic models are one form of knowledge organisation that can result from this process. Synthetic models are not stable, but dynamic and constantly changing as children's developing knowledge constantly evolves. Finally, it should be clear from the above that our theoretical position is a constructivist one, as it explains misconceptions to result from students constructive attempts to add new information onto existing but incompatible knowledge structures. Last, our approach provides a comprehensive framework within which meaningful and detailed predictions can be made about the knowledge acquisition process.

The 'knowledge in pieces' view

DiSessa (1988; 1993, this volume) has put forward a different proposal for conceptualizing the development of physical knowledge. He argues that the knowledge system of novices consists of an unstructured collection of many simple elements known as phenomenological primitives (p-prims) that originate from superficial interpretations of physical reality. P-prims are supposed to be organized in a conceptual network and to be activated through a mechanism of recognition that depends on the connections that p-prims have to the other elements of the system. According to this position, the process of learning science is one of collecting and systematizing the pieces of knowledge into larger wholes. This happens as p-prims change their function from relatively isolated, self-explanatory entities to become pieces of a larger system of complex knowledge structures such as physics laws. In the knowledge system of the expert, p-prims "can no longer be self-explanatory, but must refer to much more complex knowledge structures, physics laws, etc. for justification (diSessa, 1993, p. 114).

Our position is not inconsistent with the view that something like diSessa's p-prims constitute an element of the knowledge system of novices and experts. We believe that p-prims can be interpreted to refer to the multiplicity of perceptual and sensory experiences obtained through observations of physical objects and interactions with them. In the conceptual system we propose, diSessa's p-prims would take the place of observation-based beliefs. Our proposal that the conceptual system consists of different kinds of knowledge elements (such as beliefs, presuppositions and mental models) is also consistent with diSessa's proposal that we need to focus not on single conceptions but on rich knowledge systems composed of many constituent elements.

Disessa argues that p-prims are basically unstructured or loosely organized in the conceptual system of the novice. It is through instruction and exposure to the scientific theory that p-prims lose their self-explanatory status and become organized in larger theoretical structures such as physical laws. According to diSessa this change in the function of p-prims is a major change from "intuitive to expert physics".

In our view, (and to the extent that knowledge elements such as p-prims could be postulated to operate in our conceptual system), p-prims should become organized in knowledge structures much earlier than diSessa believes. If this is so, the process of learning science is not one of simply organizing the unstructured p-prims into physics laws but rather one during which they need to be re-organised into a scientific theory. This is a slow, gradual process, precisely because we are dealing with many knowledge elements.

Sociocultural approaches

Criticisms from sociocultural theorists point out that conceptual change is not an individual, internal, cognitive process, as it is often seen from a purely cognitive perspective. Rather, they think it should be considered as a social activity that takes place in complex sociocultural settings that also involve the use of symbolic languages, tools, and artefacts. We also believe that it is important to consider the role of sociocultural practices, tools, and contexts in problem solving and reasoning. However, this should be not be done without consideration of the crucial role individual minds play in intellectual functioning. As Hatano (1994) aptly expresses "although understanding is a social process, it also involves much processing by an active individual mind. It is unlikely that conceptual change is induced only by social consensus. The post-change conceptual systems must have not only coherence but also subjective necessity. Such a system can be built only through an individual minds' active attempts to achieve integration and plausibility" (pp. 195).

A second criticism coming from the more radical expression of sociocultural theory raises questions about the very nature of concepts and the ontological status of knowledge itself. From the point of view of sociocultural theory, knowledge is not something that can be acquired, develop, or change but "a process, an activity that takes place among individuals, the tools and artifacts that they use, and the communities and practices in which they participate" (Greeno et al., 1996, p. 20).

This position emerged in an effort by sociocultural theorists to explain the results of a set of empirical findings showing that learning is highly influenced by contextual and situational factors and that there is often a lack of knowledge transfer, usually in cases where information learned in school needs to transfer to everyday, out of school situations (see Vosniadou, 2006 for a discussion of these issues). For example, studies of math problem solving in practical situations have shown that the procedures used for problem solving at school do not transfer to math problem solving in everyday contexts (Carraher, Carraher, & Schliemann, 1985; Lave, 1988; Scribner, 1984). These findings have led some researchers to propose a highly contextualized view of knowledge as a process of participation in sociocultural activities (see also Sfard, 1998).

While it is important to recognize the problems that cognitive theory has with transfer as identified by sociocultural theorists, the move to deny any objectification of knowledge – and thus of the possibility of any transfer, does not provide a viable solution. There is an enormous body of empirical evidence demonstrating beyond any possible doubt the transfer of prior knowledge and its effects, positive or negative, on reasoning, text comprehension, language communication, problem solving, memory and the acquisition of new knowledge (see Bransford, 1979; Bransford, Brown, & Cocking, 1999, for relevant reviews).

We believe that there is a different interpretation of the results of the practical math studies, which is very much related to the problem of conceptual change. More specifically, we claim that researchers have overlooked the fact that there is an important asymmetry in knowledge transfer; i.e., that it is difficult when scientific or mathematical knowledge acquired in school settings needs to be transfered to everyday situations, but not the other way around. Knowledge acquired in everyday settings is ubiquitous and transfers spontaneously and without any difficulty. The construction of misconceptions and synthetic models is additional evidence for the existence of such knowledge transfer (Vosniadou, 2007a).

As it was earlier discussed, many science and mathematics concepts are difficult to learn because they are embedded in different explanatory frameworks from the initial, framework theories constructed by children early on in development. In these situations, it is very common for knowledge acquired in school settings either to remain unrelated to prior knowledge, or to be added to what is already known through the use of additive, enrichment mechanisms. As mentioned earlier, many misconceptions can be interpreted as synthetic models, resulting from the implicit use of bottom-up, additive learning mechanisms in situations where the new information, belongs to a different explanatory framework from that of prior knowledge. These implicit but constructive attempts are nothing more than the instances of negative transfer where prior knowledge stands in the way of understanding science and math concepts.

Such findings are not easy to explain from a sociocultural perspective, that denies any objectification of knowledge. This is the reason why some sociocultural researchers dismiss the effects of prior knowledge and the evidence for negative knowledge transfer and synthetic models altogether (e.g., Schoultz, Saljo and Wyndhamn, 2001; Nobes, et al., 2005). These researcher consider the empirical findings demonstrating the presence of misconceptions to be methodological artifacts, caused by flaws in studies conducted from a cognitive perspective that focus on unobservable, inside-the-head, mental structures. They claim that these difficulties disappear when thinking and reasoning is analyzed from a discursive point of view, as a tool dependent activity. However, the strategy to deny the empirical findings regarding misconceptions or the effects of prior knowledge in general, is totally inadequate and does not solve any problems. It is true that children produce fewer misconceptions when forced-choice questionnaires are used and cultural artefacts like a globe are present, but they still have considerable difficulty understanding science and math concepts (see Brewer, this volume; Vosniadou et al., 2004, 2005, Erhlin, 2007). Students' difficulties in learning the concepts of current science and mathematics have been documented in hundreds of studies and represent one of the most pressing problems of schooling. They are not going to disappear because they are not consistent with the radical sociocultural perspective. Rather, it is the sociocultural perspective that needs to be modified to allow for the possibility to objectify knowledge.

To sum up, in order to explain the basic empirical findings in learning research and particularly around the problem of transfer, we need to take seriously into consideration the problem of conceptual change. We need to understand the asymmetry that exists in transfer situations and the causes of this asymmetry. This necessitates an approach on concepts that neither denies their existence, like the radical sociocultural perspective, nor considers them as stable and unchanging structures. Rather, concepts should be seen as flexible and malleable structures, influenced by the surrounding context, but also developing and evolving as the larger frameworks within which they are embedded also change.

Finally a third criticism coming from radical sociocultural theory is the denial of mental representations and mental models. From our point of view human's ability to form mental representations of the environment is important because it helps in the de-situation of cognitive activity (Greeno, 1988). Not only can we form mental models of the physical environment, we can also objectify these representations further in the creation of tools and artifacts that can then be used as external, prosthetic devices in thinking. The sociocultural perspective emphasizes the importance of cultural artifacts and the role they play as facilitators of thinking. But it does not explain how human culture created these artifacts in the first place. Model-based reasoning is the key to understanding how humans created the rich cultural environments that mediate our social and intellectual life. Cultural mediating structures can range from symbolic systems like language, mathematics, reading, writing, to artifacts like pencil and paper, calculators and computers. But even traffic lights, supermarket layouts, or categorization systems can be considered as symbolic structures that mediate our activities.

Individuals can form mental models not only of their everyday, physical experiences, but also of the cultural artifacts they use. Cultural artifacts like maps and globes can be internalized and used in instrumental ways in revising representations based on everyday experience. As mentioned earlier, our studies of children's reasoning in astronomy provide important although preliminary information about how individuals can construct mental representations that are neither copies of external reality nor copies of external artifacts, but creative synthetic combinations of both. This suggests that the cognitive system is flexible and capable of utilizing a variety of external and internal representations to adapt to the needs of the situation (Vosniadou, Skopeliti & Ikospentaki, 2004, 2005).

Some Implications for Instruction

A large body of empirical evidence has been accumulated in the last 20 or so years pointing to the problem of conceptual change, particularly in the areas of science and mathematics. Nevertheless, the relevant findings and results have not yet found their way in everyday classroom practices. This is true both for science teaching (Duit, 2007; this volume) and mathematics (see also Greer, 2004, 2006; Resnick, 2006). Science and mathematics educators often believe that there is little or no prior knowledge that students bring to the learning task. Or, they believe that new concepts can always be built upon prior knowledge through enrichment types of mechanisms. They do not understand that prior knowledge can sometimes hinder the acquisition of new information.

Teaching for conceptual change requires that teachers pay attention to the prior knowledge that students bring to the learning task and finding ways not only to enrich this prior knowledge but also to change it, leading eventually to the formation of new structures. This requires the design of appropriate curricula and of instruction that aims towards the creation of motivated, life-long learners who have the necessary metaconceptual awareness and intentional learning strategies to engage in prolonged and meaningful learning (Vosniadou, Ioannides, Dimitrakopoulou & Papademetriou, 2001).

In the context of physics education, instruction for conceptual change has often been associated with the 'classical approach' (Posner et al. 1982) earlier described, based on the use of cognitive conflict. An important limitation of this type of instruction is the assumption that conceptual change is something that can happen in a short period of time and involves a rational process of concept replacement, similar to a Gestalt-type of restructuring experience. On the contrary, as we have discussed in this paper, the process of conceptual change is a gradual and continuous process that involves many interrelated pieces of knowledge and requires a long time to be achieved. Limited uses of cognitive conflict can be useful as an instructional strategy but only in the context of a larger program of carefully planned curricula and interventions.

There are a number of directions one could take in the development of instructional approaches to promote conceptual change. One direction is to try to narrow the gap between students' framework theories, and the new, to-be-acquired explanatory frameworks necessary for understanding science and mathematics concepts, thus making the problem of conceptual change less acute. This can be done by taking a long-term perspective in curricula design as well as in everyday teaching practices, by anticipating later expansions of meaning, identifying the points at which conceptual change is necessary and by looking for bridging devices from early on (Greer, 2006).

This recommendation comes in contrast to current practices where insufficient attention to the issue of conceptual change and the implicit belief that learning progresses 'along a simple/complex dimension' (Greer, 2006, p.178) results in the design of curricula that introduce 'simple' concepts first and more 'complex' concepts later. As Vosniadou & Vamvakoussi (2006) note, the concepts that are considered to be "simpler" are usually the ones closer to children's intuitive understandings. Thus, children's initial theories are confirmed and strengthened through instruction, resulting in cognitive inflexibility that widens the gap between children's current knowledge and the to-be-acquired information, hindering further understanding.

A more radical proposal has been offered by some mathematics education researchers who suggest that curricula should support the introduction of certain difficult concepts at an earlier stage. For instance, it has been proposed that instead of teaching arithmetic first and algebra second the two strands should be intertwined from an early age (see Carraher, Schliemann, & Brizuela, 2001). There are also interesting proposals about how to teach rational number in ways that can be understood by very young children (Nunes, 2007). We consider these to be interesting proposals that need to be further investigated first at the experimental level.

Regardless of how far one can go in knowledge acquisition relying on social-constructivist types of approaches that build on prior knowledge through natural, implicit, enrichment types of mechanisms, the problems of conceptual change require that teachers also teach to students mechanisms for knowledge restructuring, such as model-based reasoning, the deliberate uses of bridging analogies, and cross domain mappings. Instructional interventions should also pay attention to the development of students who have the metaconceptual awareness, epistemological sophistication and intentional learning skills that will allow them to the engage in meaningful, long-term learning (Sinatra & Pintrich, 2003; Wiser & Smith, this volume; Vosniadou, 2003).

We agree with Hatano and Inagaki (2003) that considerable social support is required for this type of instruction. One way teachers can provide the socio-cultural environment to encourage metaconceptual awareness is to ask students to participate in dialogical interaction, which is usually whole-class discussion. Whole classroom dialogue can be effective because it ensures on the one hand that students understand the need to revise their beliefs deeply instead of engaging in local repairs (Chinn & Brewer, 1993), and on the other that they spend the considerable time and effort needed to engage in the conscious and deliberate belief revision required for conceptual change (see also Miyake, 1986; this volume).

Hatano, together with his colleague Kayoko Inagaki, have conducted a number of educational studies in order to show how individual cognitive mechanisms can combine with socio-cultural constraints to promote instruction-induced conceptual change (Hatano, 1996; Hatano & Inagaki, 1991; Inagaki, Hatano & Moritas, 1998). Most of these studies are conducted using the Japanese science education method known as Hypothesis-Experiment-Instruction (HEI) originally devised by Itakura (Itakura, 1962). This method was utilized extensively by Hatano and his colleagues and it is a promising method for achieving the kind of metaconceptual awareness and intentional learning required by students for the deliberate and intentional belief revision needed for instruction based conceptual change (see also Miyake, this volume).

It is probably clear by now that teaching and learning for conceptual change requires substantial amounts of effort on the part of the teacher, as well as on the part of the learner. For this effort to be invested, there should be an environment within which this is both necessary and appreciated. That is, for teachers to design relevant and meaningful activities (Vosniadou et al., 2001), and for students to be actively engaged, there should be a broader educational community that recognizes and is capable of assessing this kind of effort.

Conclusions

We argued that science and mathematics concepts are difficult to learn because they are embedded in initial, framework theories of physics and mathematics, which are different explanatory frameworks from those that are now scientifically accepted. These naïve framework theories are not fragmented observations but form a relatively coherent explanatory system which is based and continuously re-confirmed by everyday experience. Students are not aware of these differences and employ the usual enrichment mechanisms to add scientific and mathematical information to existing knowledge structures, destroying their coherence and creating internal inconsistency and misconceptions which are 'synthetic models'.

In order to foster conceptual change through instruction, we can consider the design of curricula and instruction that reduce the gap between students' expected initial knowledge and the to-be-acquired information, so that learners can use their usual constructive, enrichment types of learning mechanisms successfully. It is also important to develop in students the necessary metaconceptual awareness, epistemological sophistication, hypothesis testing skills, and the top-down, conscious and deliberate mechanisms for intentional learning that will prepare them for meaningful, life-long learning. Instruction for conceptual change thus requires not only the restructuring of students' naïve theories but also the restructuring of their modes of learning and reasoning. The above cannot be accomplished without substantial sociocultural support.

References

Baillargeon, R., (1995). A model of physical reasoning in infancy. In *Advances in infancy research, vol. 9*. Ablex.

Barker, M. & Carr, M. (1989). Teaching and Learning about Photosynthesis. *International Journal of Science Education, 11*(1), pp. 48-56.

Behr, M., Post, T., & Lesh, R., (1981(. Construct Analyses, Manipulative Aids, Representational Systems and the Learning of Rational Numbers. In *Proceedings of the 5th Conference of the International Group for the Psychology of Mathematics Education* (pp. 203-209). Grenoble, France: PME.

Bereiter, C. (1984). How to keep thinking skills from going the way of all frills. *Educational Leadership, 42*, pp. 75-77.

Blown, E.J., & Bryce, T.G.K. (2006). Knowledge Restructuring in the Development of Children's Cosmologies. I*nternational Journal of Science Education, 28*(12), 1411–1462.

Bransford, J., (1979). *Human cognition: Learning, understanding, and remembering.* Belmont, CA: Wadsworth.

Bransford, J.D., Brown, A., & Cocking, R., (Eds.). (1999). *How people learn: Mind, brain, experience and school*. Washington, DC: National Academy Press

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1985). Mathematics in the streets and in schools. British Journal of Developmental Psychology, 3, 21- 29.

Carraher, D., Schliemann, A., & Brizuela, B. (2001). Can young students operate on unknowns? In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education* (vol.1, pp. 130-140). Utrecht: Utrecht University.

Carey, S. (1985). *Conceptual change in childhood*. Cambridge, MA: MIT Press.

Carey, S. 1991. Knowledge acquisition: Enrichment or conceptual change? In S. Carey & R. Gelman (Eds.), *The epigenisis of mind: Essays on biology and cognition.* Hillsdale, NJ: Erlbaum.

Carey, S. & Spelke, E. (1994). Domain-specific knowledge and conceptual change. In L. A. Hirschfeld and S. A. Gelman (Eds), *Mapping the Mind: Domain Specificity in Cognition and Culture*, Cambridge: Cambridge University Press.

Chase, W. G. & Simon, H. A. (1973). The mind's eye in chess. In W. G. Chase (Ed.), V*isual Information Processing* (pp. 215-281). New York: Academic Press.

Chi, M. T. H. (1992). Conceptual change within and across ontological categories: Examples from learning and discovery in science. In R. Giere (Ed.), *Cognitive Models of Science: Minnesota Studies in the Philosophy of Science*, (pp. 129-186). University of Minnesota Press: Minneapolis, MN.

Chi, M. T. H., Feltovich, P. J., & Glaser, R. (1981). Categorization and representation in physics problems by experts and novices. *Cognitive Science, 5*, pp. 121-152.

Chi, M.T.H. & Koeske, R.D. (1983). Network Representation of a Child's Dinosaur Knowledge. *Developmental Psychology 19*(1), 29-39.

Chinn, C. A., & Brewer, W. F. (1993). The role of anomalous data in knowledge acquisition: A theoretical framework and implications for science instruction. *Review of Educational Research*, *63*, 1-49.

Chomsky, N. (1988). *Language and problems of knowledge*. Cambridge, MA: MIT Press.

Clements, D. H. & McMillen, S., (1996). Rethinking 'concrete' manipulatives. *Teaching Children Mathematics, 2*(5), pp. 270-279.

Dehaene, S. (1998). *The number sense: How the mind creates mathematics*. Harmondsworth, Middlesex, England: The Penguin Press (First published by Oxford University Press, 1997).

Diakidoy, I.A., Vosniadou, S., & Hawks J. (1997). Conceptual change in Astronomy: Models of the earth and of the Day/night cycle in American-Indian children. *European Journal of Psychology of Education, XII*, 159-184.

diSessa, A. A. (1988) Knowledge in pieces. In G. Forman and P. B. Pufall (Eds.) Constructivism in the Computer Age (Hillsdale, NJ: Erlbaum), 49-7

diSessa, A. (1993). Toward an Epistemology of Physics. *Cognition and Instruction*, *10*(2/3), pp. 105-225.

Duit, R., (2007). Science Education Research Internationally: Conceptions, Research Methods, Domains of Research. *Eurasia Journal of Mathematics, Science & Technology Education, 3*(1), 3-15.

Elman, J., Bates, L., Johnson, M., Karmiloff-Smith, A., Parisi, D., & Plunkett K. (1996). *Rethinking innateness. A connectionist perspective on development*. Cambridge, Mass.: MIT Press.

Feyerabend, P. K., (1962). Explanation, reduction, and empiricism. In H. Feigl & G. Maxwell (Eds.), *Minne-sota studies in the philosophy of science (Vol. 3): Scientific explanation, space and time*. Minneapolis: University of Minnesota Press.

Fischbein, E. (1987). *Intuition in science and mathematics: An educational approach*. Dordrecht, The Netherlands: Reidel.

Fischbein, E., Deri, M., Nello, M., & Marino, M. (1985). The role of implicit models in solving problems in multiplication and division. *Journal for Research in Mathematics Education*, *16*, 3–17.

Gallistel, R. (1990). *The Organization of Learning*. Cambridge, Ma.: MIT Press.

Gelman, R. (1990). First principles organize attention to and learning about relevant data: Number and animate-inanimate distinction as examples. *Cognitive Science, 14*, pp. 79-106.

Gelman, R. (1991). Epigenetic foundations of knowledge structures: Initial and transcendent constructions. In S. Carey and R. Gelman (Eds.), *The epigenesis of mind: Essays on biology and cognition* (pp.293-322). Hilldale, NJ: Erlabaum.

Gelman, R. (1994). Constructivism and supporting environments. In D. Tirosh (Ed.), *Implicit and explicit knowledge: An educational approach* (pp. 55-82). New York: Ablex.

Gelman, R. (2000). The epigenesis of mathematical thinking. *Journal of Applied Developmental Psychology, 21*, pp. 27–37.

Goodman, N. (1972). Seven strictures on similarity. In N. Goodman (Ed.), *Problems and project*s (pp. 341--352). New York: Bobbs-Merrill.

Greeno, J. G., (1988). Situations, mental models, and generative knowledge. In D. Klahr & K. Kotovsky (Eds.), *Complex information processing* (pp. 285-318). Hillsdale, NJ: Lawrence Erlbaum.

Greeno, J.G., Collins, A.M., & Resnick, L.B., (1996). Cognition and learning. In D.C. Berliner & R.C. Calfee (Eds.), *Handbook of Educational Psychology*. New York: McMillan.

Greer, B. (2004). The growth of mathematics through conceptual restructuring. In S. Vosniadou & L. Verschaffel, *The Conceptual Change Approach to Mathematics Learning and Teaching, Special Issue of Learning and Instruction,* 14(5), 541-548.

Greer, B. (2006). Designing for conceptual change. In J. Novotná, H. Moraová, M. Krátká, &N. Stehlíková (Eds), *Proceedings of the 30th conference of the international group for the psychology of mathematics education* (Vol. 1, pp. 175–178). Prague: PME.

Greer, B., & Verschaffel, L. (2007). Nurturing conceptual change in mathematics education. In S. Vosniadou, A. Baltas, & X. Vamvakoussi (Eds.), *Reframing the conceptual change approach in learning and instruction* (pp. 319-328). Oxford: Elsevier.

Haslam, F. & Treagust, D.F., (1987). Diagnosing secondary students' misconceptions of photosynthesis and respiration in plants using a two-tier multiple choice instrument. *Journal of Biological Education, 21*(3), pp. 203-211.

Hatano, G. (Guest Editor), (1994). Introduction: Conceptual Change – Japanese Perspectives. *Special Issue of Human Development, 37*(4), pp. 189-197.

Hatano, G., (1996). A conception of knowledge acquisition and its implications for mathematics education, In Steffe, L. P. & Nesher, P., (Eds.) *Theories of Mathematical Learning*, Lawrence Erlbaum Associates, Mahwah, NJ.

Hatano, G. (2002). [Review of the book *The theory and practice of cultural-historical psychology*]. *Mind, Culture, and Activity: An International Journal, 9,* 238–240.

Hatano, G., & Inagaki, K. (1994). Young children's naive theory of biology. *Cognition, 50*, pp. 171–188.

Hatano, G., & Inagaki, K. (1991). Sharing cognition through collective comprehension activity. In L. B. Resnick, J.M. Levine & S. D. Teasley (Eds.), *Socially shared cognition* (pp. 331–348). Washington, DC: American Psychological Association.

Hatano, G. & Inagaki, K. (1997) Qualitative changes in intuitive biology. *European Journal of Psychology of Education*, Vol. *XII*, 111-130.

Hatano, G., & Inagaki, K. (2000). Domain-specific constraints on conceptual development . *International Journal of Behavioural Development, 24*(3), 267–275.

Hatano, G. & Inagaki, K. (2003). When is conceptual change intended? A cognitive-sociocultural view. In G.M. Sinatra & P.R. Pintrich (eds.). *Intentional conceptual change*, pp. 407-427, Mahwah, New Jersey: Lawrence Erlbaum Associates.

Hatano, G., & Miyake, N., (1991). What Does a Cultural Approach Offer to Research on Learning? *Learning and* Instruction, *1*, pp. 273-281.

Hayes, B. K., Goodhew, A., Heit, E., & Gillan, J. (2003). The role of diverse instruction in conceptual change. *Journal of Experimental Child Psychology, 86*, 253-276.

Inagaki, K. & Hatano, G. (2003). Conceptual and linguistic factors in inductive projection: How do young children recognize commonalities between animals and plants? In D. Gentner & S. Goldin-Meadow (Eds.), *Language in Mind* (pp. 313-333). Cambridge, MA: MIT Press.

Inagaki, K., Hatano, G. and Moritas, E. (1998). Construction of mathematical knowledge through whole class discussion, *Learning and Instruction, 8*, 503–526.

Ioannides, C., & Vosniadou, S. (2002). The changing meanings of force. *Cognitive Science Quarterly, 2*(1), 5-62.

Ioannidou, I., & Vosniadou, S., (2001). The development of knowledge about the composition and layering of the earth's interior. *Nea Paedia, 31*, pp. 107–150 (in Greek).

Itakura, K. (1962). Instruction and learning of concept force in static based on Kasetsu-Jikken-Jugyo (Hypothesis – Experiment – Instruction): A new method of science teaching. *Bulletin of National Institute for Educational Research, 52*. (in Japanese)

Keil, F. C. (1981). Constraints on knowledge and cognitive development, P*sychological Review 88*, pp. 197-227.

Keil, F. C. (1990). Constraints on constraints: Surveying the epigenetic landscape. *Cognitive Science, 14*, pp. 135-168.

Keil, F. C., (1994). *Concepts, Kinds, and Cognitive Development* Cambridge, MA: MIT Press.

Khoury, H. A., & Zazkis, R. (1994). On fractions and non-standard representations: Preservice teachers' concepts. *Educational Studies in Mathematics*, *27*, 191–204.

Kikas, E. (1998). The impact of teaching on students' definitions and explanations of astronomical phenomena. *Learning and Instruction, 8*(5), 439–454.

Kitcher, P. (1983). *The Nature of Mathematical Knowledge*, Oxford, Oxford University Press.

Kobayashi, Y. (1994). Conceptual acquisition and change through social interaction. *Human Development, 37*, pp. 233-241.

Kuhn, T. (1962). *The Structure of Scientific Revolutions*. Chicago: Chicago Press.

Kyrkos, Ch., & Vosniadou, S., (1997). *Mental models of plant nutrition: A study of conceptual change in childhood.* Poster presented at the seventh European Conference for Research on Learning and Instruction, Athens, Greece.

Lave, J., (1988). *Cognition in Practice: Mind, Mathematics and Culture in Everyday Life*, Cambridge University Press, Cambridge.

Machamer, P. (2007). Kuhn's Philosophical Successes? In S. Vosniadou, A., Baltas, & X., Vamvakoussi, (Eds.). *Reframing the conceptual change approach in learning and instruction*. (pp. 35-46), Oxford: Elsevier.

Malara, N. (2001). From fractions to rational numbers in their structure: Outlines for an innovative didactical strategy and the question of density. In: J. Novotná (Ed.), *Proceedings of the 2nd Conference of the European Society for Research Mathematics Education II* (pp. 35–46). Praga: Univerzita Karlova v Praze, Pedagogická Faculta.

Mali, G. B. & Howe, A. (1979). Development of Earth and gravity concepts among Nepali children. *Science Education, 63*(5), pp. 685-691.

Markman, E. M. (1989). *Categorization and Naming in Children: Problems of Induction*. MIT Press.

Markovitz, Z., & Sowder, J. (1991). Students' understanding of the relationship between fractions and decimals. *Focus on Learning Problems in Mathematics*, *13*(1), 3–11.

Maria, K. (1993). *The Development of Earth Concepts.* Paper presented at the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics, Ithaca, NY.

Maria, K. (1997a). A case study of conceptual change in a young child. *Elementary School Journal, 98*, 67-88.

Maria, K. (1997b). *Conceptual change in a young girl: A longitudinal case study*. Paper presented at the annual meeting of the National Reading Conference, Scottsdale, AZ.

Mason, L. (2003). High school students' beliefs about maths, mathematical problem solving and their achievement in maths: a cross-sectional study. *Educational Psychology, 23*(1), pp. 73-85.

Mayer, R. E. (1985). Implications of cognitive psychology for instruction in mathematical problem solving. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 123-138). Hillsdale, NJ: Lawrence Erlbaum Associates.

McCloskey, M. (1983 a). Naive Theories of Motion. In D. Gentner & A. Stevens (Eds.), *Mental Models*, Hillsdale: Erlbaum.

McCloskey, M. (1983 b). Intuitive Physics. *Scientific American, 248*(4), pp. 122-130.

Medin, D.L. & Rips, L.J. (2005) Concepts and categories: memory, meaning, and metaphysics. In Holyoak, K.J. & Morrison, R.G. (Eds) *Cambridge Handbook of Thinking and Reasoning* (pp. 37–72). Cambridge: Cambridge University Press.

Merenluoto, K., & Lehtinen, E. (2002). Conceptual change in mathematics: Understanding the real numbers. In: M. Limon, & L. Mason (Eds), *Reconsidering conceptual change: Issues in theory and practice* (pp. 233–258). Dordrecht, The Netherlands: Kluwer.

Merenluoto, K., & Lehtinen, E. (2004). Number concept and conceptual change: Outlines for new teaching strategies. *Learning and Instruction*, *14*, 519–534.

Miyake, N. (1986) Constructive Interaction and the Iterative Process of Understanding. *Cognitive Science*, 10, 151-177.

Moskal, B. M., & Magone, M. E. (2000). Making sense of what students know: Examining the referents, relationships and modes students displayed in response to a decimal task. *Educational Studies in Mathematics*, *43*, 313–335.

Moss, D., (2005). Magnetic fields in A stars. In J. Zverko, J. Ziznovsky, S.J. Adelman, & W.W. Weiss, (Eds.) *Proceedings of the International Astronomical Union Symposium No. 224* (pp. 245-252). Cambridge University Press

Nersessian, N. (1992). How do scientists think? Capturing the dynamics of conceptual change in science. In R. N. Giere (Ed.), *Minnesota studies in philosophy of science: Vo/.* 15. *Cognitive models of science.* Minneapolis: University of Minnesota Press.

Neumann, R. (1998). Students' ideas on the density of fractions. In: H. G. Weigand, A. Peter-Koop, N. Neil, K. Reiss, G. Torner, & B. Wollring (Eds), *Proceedings of the annual meeting of the Gesellschaft fur Didaktik der Mathematik* (pp. 97–104). Retrieved March 21, 2005 from http://webdoc.sub.gwdg.de/ebook/e/gdm/1998/

Ni, Y., & Zhou, Y-D. (2005). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist*, *40*(1), 27–52.

Nobes, G., Martin, A.E., & Panagiotaki, G., (2005). The development of scientific knowledge of the Earth. *British Journal of Developmental Psychology, 23*, pp. 47-66.

Novak, J.D., (Ed.), (1987). *Proceedings of the Second International Seminar: Misconceptions and Educational Strategies in Science and Mathematics, vol. I, II, & III*. Ithaca, N.Y.: Cornell University, Department of Education.

Nunes, T. (2007). *Understanding Rational Numbers*, Invited Talk at the 12th European Conference for Research on Learning and Instruction, Budapest, Hungary.

Nussbaum, J. (1979). Children's conception of the earth as a cosmic body: A cross-age study. *Scientific Education, 63*, pp. 83-

Nussbaum, J. (1985). The earth as a cosmic body. In R. Driver, E. Guesne & A. Tiberghien (Eds.), *Children's Ideas in Science*. Open University Press, Milton Keynes.

Nussbaum, J., & Novak, J. D. (1976). An assessment of children's concepts of the earth utilizing structured interviews. *Science Education, 60*, 535–550.

O'Connor, M. C. (2001). "Can any fraction be turned into a decimal?" A case study of a mathematical group discussion. *Educational Studies in Mathematics*, *46*, 143–185.

Opfer, J. E., & Siegler, R. S. (2004). Revisiting preschoolers' *living things* concept: A microgenetic analysis of conceptual change in basic biology. *Cognitive Psychology, 49,* 301-332.

Piajet, J. (1970). *Structuralism*. New York: Basic Books.

Pintrich, P. R. (2000). The role of goal orientation in self-regulated learning. In M. Boekaerts, P. R. Pintrich, & M. Zeidner, *Handbook of self-regulation.* San Diego, CA: Academic Press.

Posner, G. J., Strike, K. A., Hewson, P. W., & Gertzog, W. A. (1982). Accommodation of a scientific conception: towards a theory of conceptual change. *Science Education, 66*, pp. 211–227.

Putnam, H. (1992). Truth, Activation Vectors and Posession Conditions for Concepts. *Philosophy and Phenomenological Research, 52*, pp. 431-447.

Resnick, L.B. (1986). The development of mathematicsl intuition. In M. Perimutter (Ed.), *Perspectives on intellectual development: The Minnesota Symposia on child Psychology* (Vol.19, pp.159-194). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Resnick, L. B. (1989). Developing mathematical knowledge. *American Psychologist*, *44*(2), 162–169.

Resnick, L.B. (2006). The dilemma of mathematical intuition in learning. In J. Novotná, H. Moraová, M. Krátká, &N. Stehlíková (Eds), *Proceedings of the 30th conference of the international group for the psychology of mathematics education* (Vol. 1, pp.173-175). Prague: PME.

Rogoff, B., (1990). *Apprenticeship in thinking: Cognitive development in social context.* New York: Oxford University Press.

Samarapungavan, A., Vosniadou, S., & Brewer, W.F. (1996). Mental models of the earth, sun, and moon: Indian children's cosmologies. *Cognitive Development, 11*, 491-521.

Schoultz, J., Säljö, R., & Wyndhamn, J., (2001). Heavenly talk: discourse, artifacts, and children's understanding of elementary astronomy. *Human Development, 44*, pp. 103-118.

Scribner, S., (1984), "Studying Working intelligence," in B, Rogoff and J. Lave (Eds.), *Everyday Cognition: Its Development in Social Context,* Cambridge, MA: Harvard University Press.

Sfard, A. (1998). On two metaphors for learning and on the dangers of choosing just one, *Educational Researcher 27*(2), 4-13.

Siegal, M., Butterworth, G., & Newcombe, P.A. (2004). Culture and children's cosmology, *Developmental Science, 7*(3), pp. 308-324.

Sinatra, G. M., and Pintrich, P. R., Eds. (2003) *Intentional conceptual change*. Mahwah, NJ: Erlbaum.

Skopeliti, I., & Vosniadou, S. (2006, July). The Influence of Refutational Texts on Children's Ideas about the Earth. In R. Sun & N. Miyake (Eds.) Proceedings of the 28th Annual Conference of the Cognitive Science Society (pp. 2608), Vancouver, Canada.

Skopeliti, I. & Vosniadou, S. (2007). The Influence of Refutational & Categorical Information on Children's Scientific Understanding. In S. H. Broughton (Chair) Exploring the Nature of the Refutation Text Effect in Conceptual Change, Symposium conducted at the 12th European Conference for Research on Learning and Instruction, Budapest, Hungary

Smith, C., Carey, S., & Wiser, M. (1985). On differentiation: A case study of the development of the concepts of size, weight, and density. *Cognition,* 21, 177-237.

Smith, J.P., diSessa, A.A., & Roschelle, J. (1993). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *The Journal of the Learning Sciences, 3*, pp. 115-163.

Smith, C. L., Solomon, G. E. A., & Carey, S. (2005). Never getting to zero: Elementary school students'understanding of the infinite divisibility of number and matter. *Cognitive Psychology*, 51, 101–140.

Spelke, E.S., (1991). Physical knowledge in infancy: Reflections on Piaget's theory. In S. Carey, & R. Gelman (Eds). *The Epigenesis of Mind: Essays on Biology and Cognition*. Lawrence Erlbaum Associates.

Stafylidou, S., & Vosniadou, S. (2004). Students' understanding of the numerical value of fractions: A conceptual change approach. *Learning and Instruction*, *14*, 503–518.

Thagard, P. (1988). *Computational Philosophies of Science*. A Bradford Book. MIT Press, Cambridge, Massachusetts.

Thagard, P. (1992). *Conceptual revolutions*. Princeton, NJ: Princeton University Press.

Tirosh, D., Fischbein, E., Graeber, A. O., & Wilson, J. W. (1999). *Prospective elementary teachers'conceptions of rational numbers.* Retrieved May 05, 2005 from http://jwilson.coe.uga.edu/Texts.Folder/Tirosh/Pros.El.Tchrs.html

Vamvakoussi, X., & Vosniadou, S. (2004). Understanding the structure of the set of rational numbers: A conceptual change approach. *Learning and Instruction*, *14*, 453–467.

Vamvakoussi, X., & Vosniadou, S. (2007). How many numbers are there in a rational numbers interval? Constraints, synthetic models and the effect of the number line. In S. Vosniadou, A. Baltas, & X.Vamvakoussi (Eds.), *Reframing the conceptual change approach in learning and instruction* (pp.265-282). Oxford: Elsevier

VanLehn, K. (1990). *Mind bugs: The origins of procedural misconceptions*. Cambridge, MA: MIT Press.

Verschaffel, L., & Vosniadou, S. (Eds.). (2004). The conceptual change approach to mathematics learning and teaching. *Learning and Instruction, 14*, 445–548.

Vosniadou, S., (1989). Analogical Reasoning and Knowledge Acquisition: A Developmental Perspective. In S. Vosniadou & A. Ortony (Eds.), *Similarity and analogical reasoning*, (pp. 413-422). New York: Cambridge University Press.

Vosniadou, S. (2000). Conceptual Change Research and the Teaching of Science. In H. Behrendt, H. Dahnck, R. Duit, W. Graber, M. Komorek, A. Kross & P. Reiska (Eds.), *Research in Science Education: Past, Present and Future*. Kluwer Academic Publishers, 177-188.

Vosniadou, S. (2003). Exploring the relationships between conceptual change and intentional learning. In G. M. Sinatra & P. R. Pintrich (Eds.), *Intentional conceptual change* (pp. 377-406). Mahwah, NJ: Lawrence Erlbaum Associates.

Vosniadou, S. (2006). The Conceptual Change Approach in the Learning and Teaching of Mathematics: An introduction. In Novotná, J., Moraová, H., Krátká, M. & Stehlíková, N. (Eds.), *Proceedings, 30th Conference of the International Group for the Psychology of Mathematics Education, Vol. 1*, pp. 155-159. Prague: PME.

Vosniadou, S. (2007 a). The Cognitive—Situative Divide and the Problem of Conceptual Change *Educational Psychologist, 42*(1), pp. 55-66.

Vosniadou, S., (2007 b). The conceptual change approach and its re-framing. In S. Vosniadou, A., Baltas, , & X., Vamvakoussi, (Eds.). *Reframing the conceptual change approach in learning and instruction*. Oxford: Elsevier.

Vosniadou, S., (2007 c). Conceptual Change and Education. *Human Development, 50*(1), pp. 47-54

Vosniadou, S., Baltas, A., & Vamvakoussi, X. (Eds.). *Reframing the conceptual change approach in learning and instruction*. Oxford: Elsevier.

Vosniadou, S. & Brewer, W.F., (1992). Mental models of the earth: A study of conceptual change in childhood. *Cognitive Psychology, 24*, pp. 535-585.

Vosniadou, S. & Brewer, W.F. (1994). Mental models of the day/night cycle. *Cognitive Science, 18*, pp. 123-183.

Vosniadou, S., Ioannides, C., Dimitrakopoulou, A., & Papademetriou, E. (2001). Designing learning environments to promote conceptual change in science. *Learning and Instruction*, *11*, pp. 381–419.

Vosniadou, S. & Skopeliti, I. (2005). Developmental Shifts in Children's Categorization of the Earth, In B. G. Bara, L.Barsalou, & M. Bucciarelli (Eds.) *Proceedings of the XXVII Annual Conference of the Cognitive Science Society*, pp. 2325-2330.

Vosniadou, S., Skopeliti, I., & Ikospentaki, K., (2004). Modes of knowing and ways of reasoning in elementary astronomy, *Cognitive Development, 19*, pp. 203-222.

Vosniadou, S., Skopeliti, I., & Ikospentaki, K., (2005). Reconsidering the Role of Artifacts in Reasoning: Children's Understanding of the Globe as a Model of the Earth, *Learning and Instruction, 15,* pp. 333-351.

Vosniadou, S., & Vamvakoussi, X. (2006). Examining mathematics learning from a conceptual change point of view: Implications for the design of learning environments. In L. Verschaffel, F. Dochy, M. Boekaerts and S.Vosniadou (Eds.), *Instructional psychology: Past, present and future trends - Fifteen essays in honour of Erik De Corte* (pp. 55-72). Oxford: Elsevier.

Vosniadou, S. & Verschaffel, L., (2004). Extending the Conceptual Change Approach to Mathematics Learning and Teaching. In L., Verschaffel and S. Vosniadou (Guest Editors), Conceptual Change in Mathematics Learning and Teaching, *Special Issue of Learning and Instruction, 14*(5), pp. 445-451.

Vygotsky, L.S., (1978). *Mind and society: The development of higher mental processes*. Cambridge, MA: Harvard University Press.

Wandersee, J.H. (1983). Students' misconceptions about photosynthesis: A cross-age study. In H. Helm & J.D. Novak (Eds.) *Proceedings of the International Seminar: Misconceptions in Science and Mathematics*, (pp. 441-446), Cornell University, Ithaca, N:Y.

Wellman, H. M. (1990). *The child's theory of mind.* Cambridge, MA: MIT Press.

Wellman, H.M., & Gelman, S.A. (1998). Knowledge acquisition in foundational domains. In D. Kuhn & R. Siegler (Eds.), *Cognition, perception and language. Volume 2 of the Handbook of child psychology*, 5th edn. (pp. 523–573). New York: Wiley.

Wiser, M., & Carey, S. (1983). When heat and temperature were one. In D. Gentner & A. Stevens (Eds.), *Mental models* (pp. 267–297). Hillsdale, NH: Erlbaum.

Table 1. Concept of the Earth

Initial	Scientific
Earth as a physical object	**Earth as an astronomical object (planet)**
Flat	Spherical
Stationary	Rotating around its axis
	Revolving around Sun
Supported	Unsupported
Up/down Gravity	Gravity towards the center of earth
Geocentric system	Heliocentric system

Table 2. Frequencies/Percentages of Children's Response Types to the Categorization Test

Questions	**Response Types**	**1st grade**	**5th grade**
1. I want you to put together the things that you think should go together, i.e. belong to the same group.	1. Distinguishes solar from physical objects and groups Earth with solar. (Two categories.)	21%	47%
	2. Distinguishes solar from physical objects and groups Earth with solar. (Many categories.)	13%	32%
	3. Distinguishes solar from physical objects and groups Earth with physical. (Many categories.)	20%	5%
	4. Does not distinguish physical from solar objects.	44%	16%
	5. Don't know.	2%	
2. Could you make only two groups from these things?	1. Distinguishes solar from physical objects and groups Earth with solar.	27%	79%
	2. Distinguishes solar from physical objects and groups Earth with physical.	11%	5%
	3. Does not distinguish physical from solar objects.	36%	16%
	4. Don't know / Cannot do it	16%	
3. Could you put in one group the things that go with the EARTH and in another the things that do not?	1. Distinguishes solar from physical objects and groups Earth with solar.	42%	90%
	2. Distinguishes solar from physical objects and groups Earth with physical.	35%	10%
	3. Does not distinguish physical from solar objects.	23%	-----

Table 3. Concept of the Number

Children's number concept (before they are exposed to rational number instruction)	*The mathematical concept of rational number*
▪ Numbers are *counting numbers*	▪ Not based on counting
▪ Numbers are discrete: There is no other number between a number and its next ▪ There is a smallest number (0 or 1)	▪ Dense: Between any two, non equal numbers there are infinitely many numbers ▪ There is no smallest numbers
▪ Numbers can be ordered by means of their position in the count list ▪ "Longer" numbers (i.e. with more digits) are bigger	▪ Ordering is not counting-based ▪ "Longer" numbers are not necessarily bigger, e.g. 3.2> 3.197
Addition and multiplication "make bigger" ▪ Subtraction and division "make smaller"	The magnitude of the outcome depends on the numbers involved, e.g. ▪ Adding a negative number "makes smaller" ▪ Multiplying by a number smaller than one "makes smaller"
▪ Every number has only one symbolic representation	▪ Any number can be represented either as a fraction, or as a decimal. In addition, any number can be represented in various ways as a fraction or decimal.

Table 4. Synthetic Models of rational numbers intervals

Category type	Models	7th grade (N=181)	9th grade (N=166)	11th grade (N=202)	Total
FIN (N=185)	All Finite Initial - Model	19 (25.0%)	21 (42.9%)	15 (25.0%)	55 (29.7%)
	All or mostly Finite for decimals and fractions	20 (26.3%)	10 (20.4%)	11 (18.3%)	41 (22.2%)
	Infinite for integers, mostly Finite for decimals and fractions	16 (21.1%)	5 (10.2%)	15 (25.0%)	36 (19.5%)
	Mixed1	21 (27.6%)	13 (26.5%)	19 (31.7%)	53 (28.6%)
	Total by Level1	76	49	60	185
FIN / INF (N=160)	Infinite for Integers, mostly Finite for decimals & fractions	2 (2.9%)	2 (4.8%)	5 (10.0%)	9 (5.6%)
	Mostly infinite for decimals, mostly finite for fractions	23 (33.8%)	6 (14.3%)	10 (20%)	39 (24.4%)
	Mostly finite for decimals, mostly infinite for fractions	8 (11.8%)	3 (7.1%)	10 (20%)	21 (13.1%)
	Mixed2	35 (51.5%)	31 (73.8%)	25 (50.0%)	91 (56.9%)
	Total by Level2	68	42	50	160
INF (N=204)	Mostly infinite for decimals, mostly finite for fractions	1 (2.7%)	3 (4%)	4 (4.3%)	8 (3.9%)
	Mostly finite for decimals, mostly infinite for fractions	0 (0%)	0 (0%)	2 (2.2%)	2 (1%)
	All Infinite -Advanced Model	3 (8.1%)	5 (6.7%)	12 (13.0%)	20 (9.8%)
	All Infinite- Sophisticated Model	5 (13.5%)	33 (44.0%)	38 (41.3%)	76 (37.3%)
	Mixed3	28 (75.7%)	34 (45.3%)	36 (39.1%)	98 (48%)
	Total by Level3	37	75	92	204

Table 5. Synthetic Models in photosynthesis (Kyrkos & Vosniadou, 1997)

1. Initial Explanation	Plants take food from the ground, through their roots. Food accumulates inside the plant and makes it grow. They do not breathe.
2. Photosynthesis as breathing, separate from feeding	Photosynthesis is about breathing and it does not affect the initial explanation of feeding. Plants take in dirty air, clean it, and they give out clean air.
3. Photosynthesis as a feeding process	Plants take food from the ground and from water through their roots. They also take food from the air and light through their leaves (O, CO_2).
4. Photosynthesis as a revised process of feeding	Plants take food from the ground and from atmosphere and also use water and O or CO_2 to make the food in their leaves through the process of photosynthesis (mixture/not a chemical process).

Table 6. Explanations of Plant Development

Plant Development (Photosynthesis)	
Initial Explanation	**Scientific Explanation**
Plants take their food from the ground (water or other nutrients) through their roots	Plants create their own food through the process of photosynthesis
Plants grow as food accumulates in small pieces inside them	Photosynthesis is a chemical process during which solar energy is used to transform water + CO_2 into organic materials like glucose. Oxygen is also formed and stored in the plant or released in the atmosphere
Plants do not breathe.	Plants take in CO_2 from the atmosphere and use it in the process of photosynthesis. To this extent "breathing" in plants is related to growth and development.

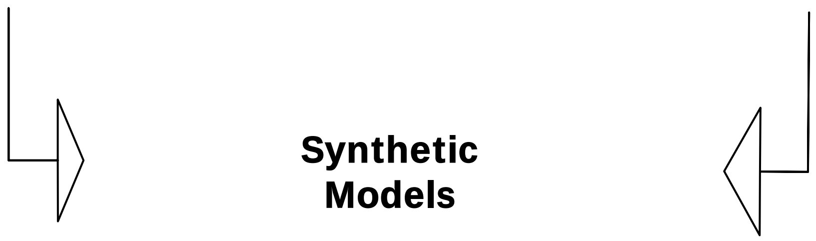


Figure 1. Mental Models of the Earth

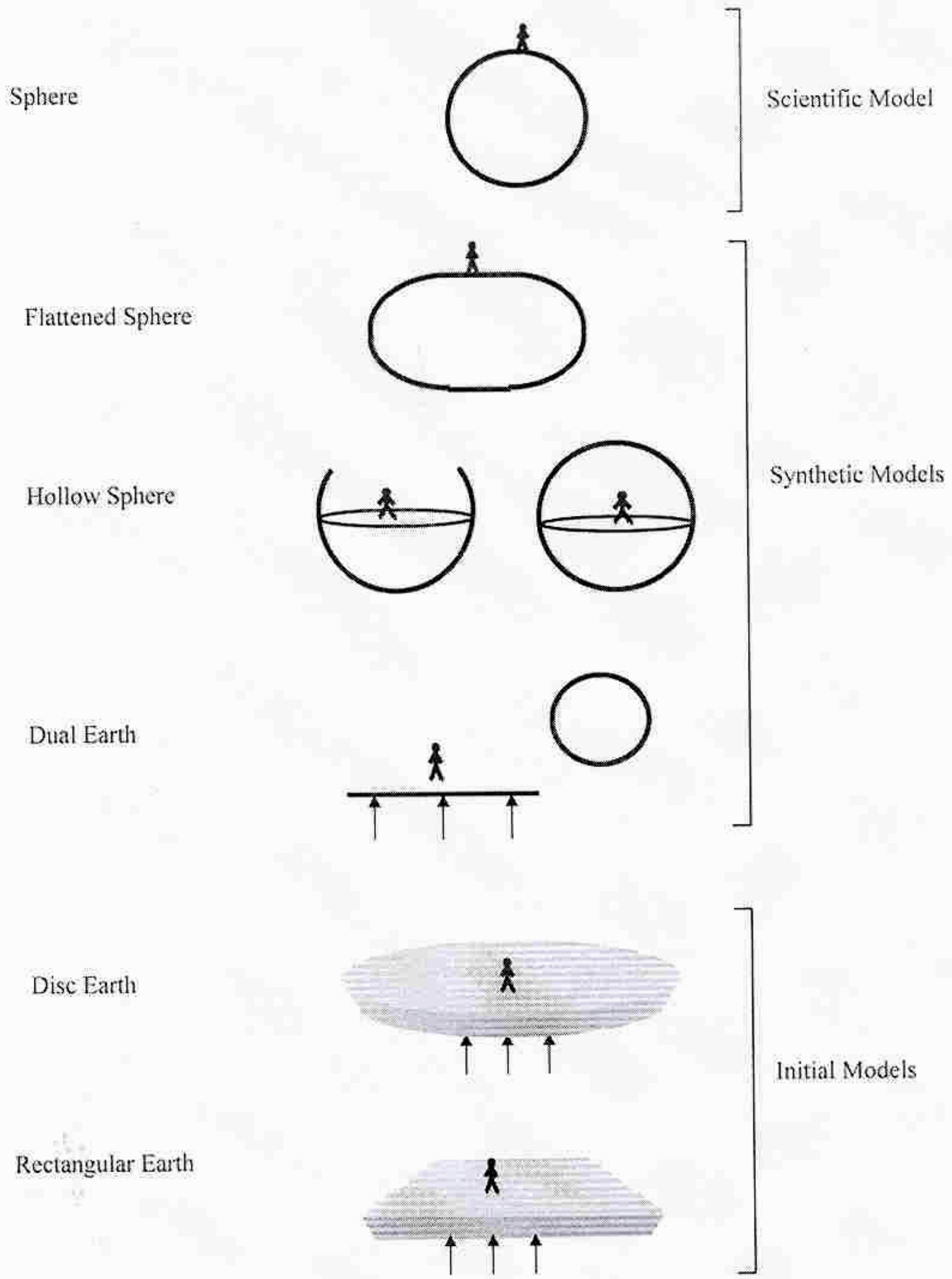


Figure 2: Hypothetical Conceptual Structure Underlying Children's Mental Models of the Earth

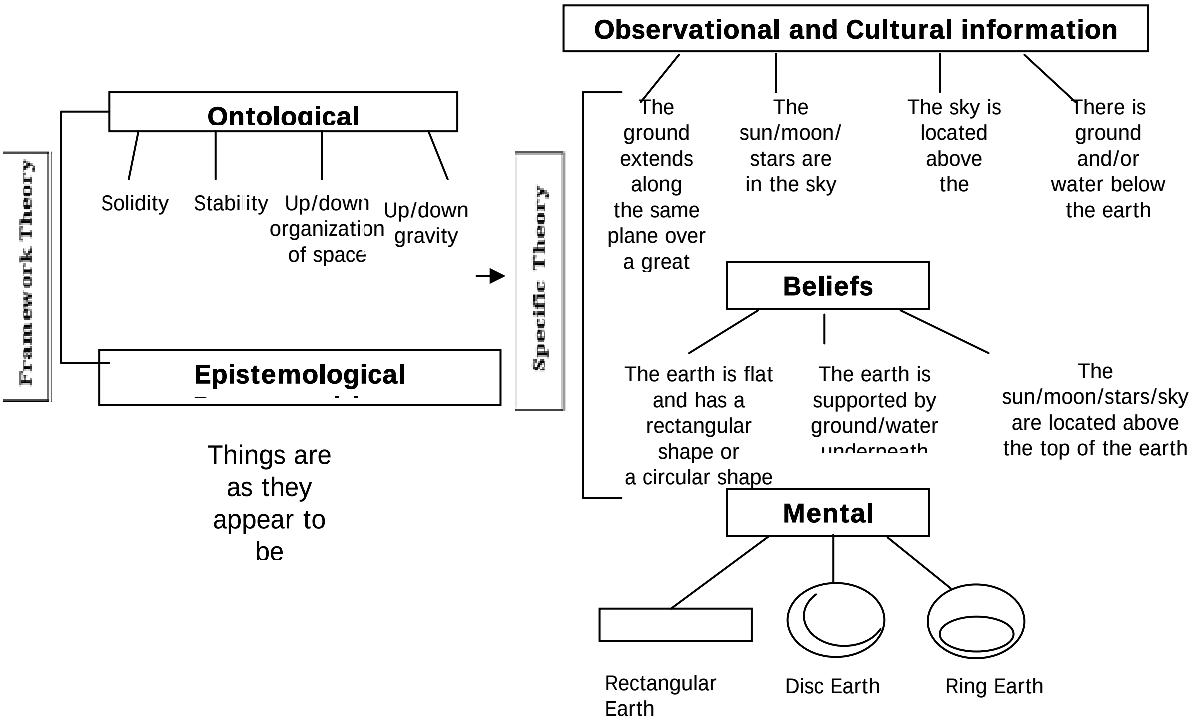


Figure 3. Percent of participants placed in Categories FIN, FIN/INF, INF as a function of grade

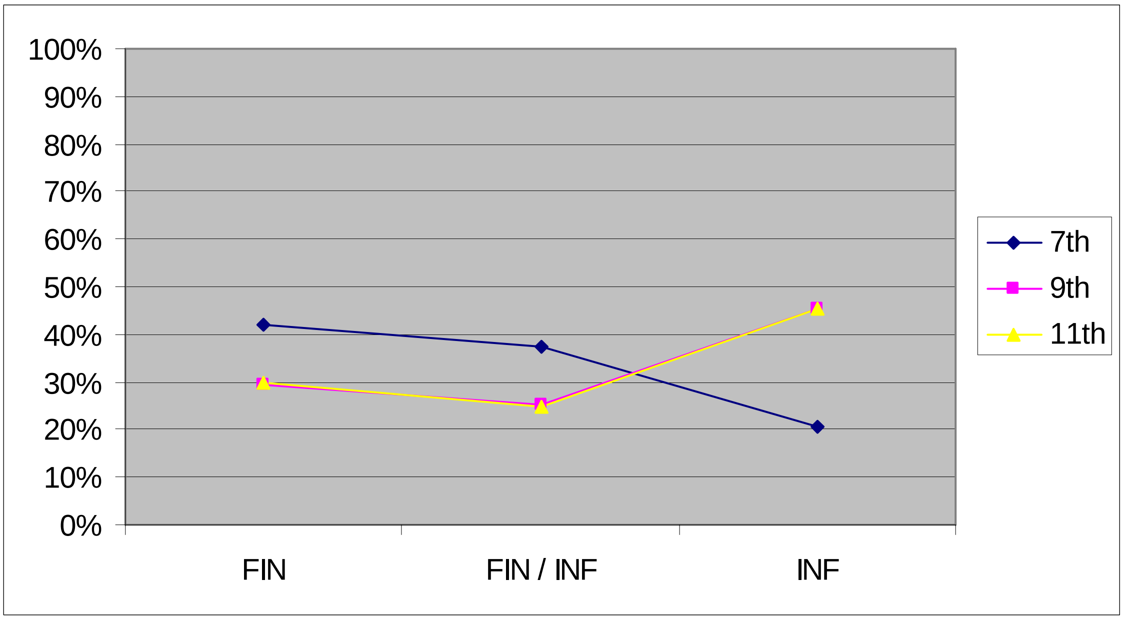
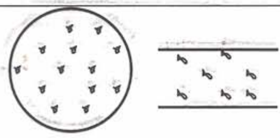
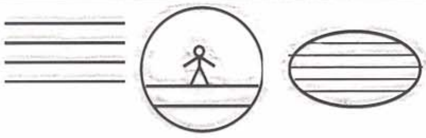


Figure 4. Frequency of Models of the Layers & Composistion of the Earth by Grade

		1st Grade	6th Grade	11th Grade
Spherical Layers *Magma in the layer below the surface*		0	1	1
Spherical Layers *Magma in the center of the earth*		2	15	15 12
Spherical Layers *Solid Materials*		2	3	6
Flat Layers *Magma in different places inside the earth*		0	0	0
Flat Layers *Magma in the bottom of the earth*		2	0	2
Undetermined Layers *Solid Materials*		4	0	0
Flat Layers *Solid Materials*		14	14	3
	Total	24	24	24