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# PROCEEDINGS

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# DEVELOPING MATHEMATICAL CREATIVITY IN PRE-SCHOOL EDUCATION

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***Abstract.*** *Could creative problem solving be the object of work in pre-school education? This study followed the work of fifteen, four and five year old children and their teacher during a two month process of solving combinatorial problems with a large number of solutions. Findings show that all children responded positively to the problems, were successful in solving them and developed sophisticated strategies during the process.*

*Key words: problem solving, fluency, flexibility, originality, elaboration.*

## THEORETICAL FRAMEWORK

Guilford (1959, 1967) describes creativity as a dynamic mental process including both divergent and convergent thinking. He goes on to describe the four components for divergent thinking as fluency-the ability to generate multiple ideas for solving a problem, flexibility-the ability to generate a variety of ideas concerning a single problem, originality-the ability to generate novel ideas and, elaboration- the ability to describe, extend and develop an idea. Of course when talking about young children we do not expected of them to create something new and of significance to the entire society or its individuals. Generally speaking, it is almost impossible for young children to create anything new (Kudryavstev, 2011). What can be expected, in terms of creativity in young children, is for the children to rediscover mathematics and reproduce its essential features- thus acting as novice mathematicians.

There is a large body of literature linking mathematical creativity to problem solving - especially mathematical problems with multiple solutions (Chamberlin and Moon, 2005; Elwood, 2009; Plucker et al, 2004; Levav-Waynberg and Leikin, 2012). At the pre-school level such mathematical problems can be problems which can be approached in different ways (Shiakalli and Zacharos, 2012; Shiakalli et al 2015), as well as mathematical problems which have a number of solutions greater than one (Shiakalli and Zacharos, 2014).

Kaufman and Sternberg (2006) note that fostering creativity depends powerfully on the learning environments while Haylock (1987) proposes that it is the role of the teacher to identify, encourage and improve creative mathematical thinking at all levels of education. More recently, Neumann (2007) underlines the importance of an interactive learning environment in the development of mathematical creativity.

In this paper we present the work of fifteen pre-school children (aged 4-5½, of a rural public Cyprus Pre-School setting) while working on combinatory mathematical problems. By closely following their work we seek to answer the questions whether children as young as 4 and 5 years old are able to solve complex combinatorial mathematical problems with a large number of solutions.

## METHOD

This study formed a part of a broader educational programme, an action research teacher professional development programme, which extended throughout the school year (October 2014- June 2015). It included the development of educational activities aimed at creating investigative learning environments in mathematics through structured lesson sessions and "Free and Structured Play Time" (Free and Structured Play Time is daily from 07:45-09:05. During this time children are free to choose and participate in playful activities aiming at developing cognitive, social, emotional and kinetic skills and abilities).

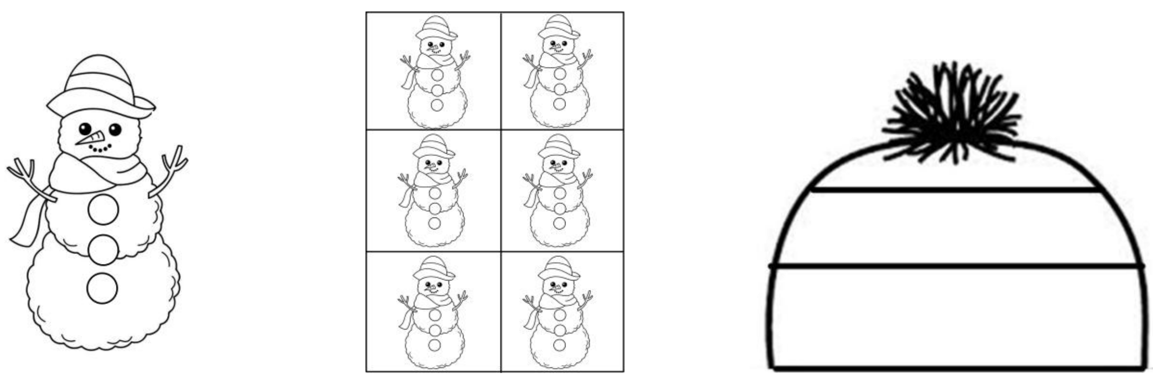
By developing the three combinatorial mathematical problems (presented in this paper) teacher and researcher anticipated that, while repeating the combinatorial problem solving process, the children would develop strategies for: (i) finding original solutions, (ii) developing fluency and flexibility in creating new solutions, and (iii) elaborating on discovered solutions in order to find new ones. Data was collected through a) videos of the teaching interventions and children's work at the Mathematics Table during "Free and Structured Play Time" (which were later analyzed by teacher and researcher based on an observation grid developed for the purposes of this study) , b) researcher field notes and c) teacher's reflective diary .

## THE MATHEMATICAL PROBLEMS

Children were asked to find all possible solutions to three combinatorial problems (the problems, manipulatives and graphical representation material are described in Table 1). The permutation without repetition problem was introduced first, and remained at the Mathematics Table during "Free and Structured Play Tine" for a period of three weeks. During this period children could chose to solve the problem as many times as they wanted (Table 2). In order for the problem to be solved, all six solutions had to be detected and graphically represented. After the permutation without repetition problem, the permutation with repetition problem was introduced. The teacher introduced the problem during Circle Time encouraging children to compare the two combinatorial problems, note similarities and differences and predict the number of solutions. The second permutation with repetition problem was set by the children: they were asked if they wanted to solve another combinatorial problem with even more solutions, were given the basic scenario "Snowy likes to colour hats" and through conversation with the teacher created the problem (Table 1). Again the children were encouraged to compare the three problems and this time (having had the second problem experience) estimate the number of solutions. The children worked on the two permutation with repetition problems in a similar way: a table was set for this task where up to four children could work simultaneously during "Free and Structured Play Time" containing coloured pencils and printed cards showing a snowman/hat (only one figure printed per card), a printed figure of a snowman / hat and buttons (in the case of the hat the buttons represented the colour each section of the hat would be coloured into). The children would use the manipulatives to create a solution. If the solution was original they would go on to graphically represent it (colour in a card) and place it on the wall. All original solutions were placed next to each other on the wall so that children could easily compare their solution to the ones already detected. The first permutation with repetition problem was solved within twelve days while the second permutation was given fifteen days. After each period, respectively, during Circle Time all solutions were placed at the centre of the circle and children were encouraged to talk about them and group them. Grouping the second problem solutions helped the children detect the solutions they had not found.

| Mathematical Problem Description | Manipulatives | Graphical Representation Material |
|---|---|---|
| *Permutation without repetition:* "Snowy has three buttons, each of a different colour. In which different ways can he place them on his tummy?" (6 solutions) | A drawing of Snowy (Figure 1a), a box of buttons | Answer sheet (Figure 1b), coloured pencils |
| *Permutation with repetition-1:* "Snowy found a box with red, green and yellow buttons. In which different ways can he place three buttons on his tummy?" (27 solutions) | A drawing of Snowy (Figure 1a), red, green and yellow buttons. | Answer cards (only one solution represented per card), coloured pencils |
| *Permutation with repetition-2:* "Snowy likes colouring hats. He has red, blue, green and yellow pencils. In which different ways can he colour his hats?" (64 solutions) | A drawing of the hat (Figure 1c), red, green, yellow, blue buttons (representing the colour of each hat section). | Answer cards (only one solution represented per card), coloured pencils |

Table 1: The mathematical problems, manipulatives and graphical representation material.



(a) (b) (c)

Figure 1: Mathematical problem manipulatives and answer sheet for the first problem.

**RESULTS**

All children chose to work on the initial permutation without repetition problem more than once (Table 2). During the repetition of the process children were observed to have used and developed different strategies for solving the problem.

| Subject and Age | Times problem was solved |
|---|---|
| S1(4), S4 (4,5), S5 (4,5), S9 (5,1), S15 (5,6) | 2 |
| S3(4,4), S8(5), S10(5,2), S12(5,4), S13 (5,4) | 3 |

| | |
|---|---|
| S7 (4,9), S11 (5,3) | 4 |
| S14 (5,5) | 5 |
| S2 (4,2), S6 (4,7), | 6 |

Table 2: Times each child chose to solve the first mathematical problem.

During their work all children were observed to had created and applied a strategy in order to solve the first combinatorial problem (Table 3). Children's work analysis showed that, not only were they able to develop and apply a strategy but they also refined it in future applications of the mathematical problem solving process.

| Strategy | Strategy Description | Subjects |
|---|---|---|
| *Detecting solutions randomly* | Placement of buttons in different places randomly and checking the answer sheet in order to avoid graphical representation of same solutions. | S1, S2, S3, S4, S5,S14 |
| Combination of deliberate alternation of colours and random placement | Placement of a different colour in fist button position and random placement of two remaining buttons in second and third position, for detection of three fist solutions. Random placement of buttons and checking answer sheet to avoid representation of same solutions- remaining sic solutions | S1, S2, S4, S5, S6, S8, S9, S10, S11, S12,S14, S15 |
| *Detection of solutions per noncontiguous pairs* | Placement of different colour in first place and random placement of colours in second and third place for identification of first three solutions. Return to a recorded solution placement of same colour in first place and reversal of colours in second and third places thus creating noncontiguous pairs of solutions. | S3, S6, S7, S8, S10, S11, S12, S13, S15 |
| *Detection of solutions per co contiguous pairs* | Detection of first solution with random placement of three colours. Detection of second solution with placement of the same colour in the first position and reversal of colours in second and third positions (creation of a pair of solutions). A change of colour in the first position and random placement of colours in second and third position. Then a reversal of colours in second and third positions (creation of second pair of solutions). Same process for the creation of the third pair of solutions. | S3, S6, S7,S10, S11, S13, S14 |

Table 3: Strategies developed during the initial problem solving process.

While working on the two permutations with repetition problems, the children showed to had developed persistence and patience as well as a positive attitude towards error. The refinement of strategies suggested elaboration while their ability to detect new solutions suggested fluency, flexibility and originality.

During the process of solving all three combinatorial problems the teacher would randomly come close to the children and encourage them to talk about their work, explain what they were doing , what they had already done and how they were planning to continue their work. When she saw that a child was finding difficulties in continuing the process (especially during the first problem solving process) she would sit next to them and work with them either by offering a helping idea or by posing helpful questions. When a child completed their work they would often call the teacher to show her their answer sheets. The teacher always reacted with enthusiasm and would express her surprise about the child's accomplishment. At the end of each process the teacher would ask the child how they felt, what they liked about the process and what they found difficult during the process.

**DISCUSSION**

In the present study we looked at pre-school children's ability to solve complex combinatorial problems with the use of graphical representations. Concerning our research question, whether pre-school children can successfully solve complex combinatorial mathematical problems with a large number of solutions, our findings show that young children apply the mathematical problem solving process in order to solve such complex mathematical problems. We think that different factors might have been influential. Firstly, the gradual development of the problems during a long period of time which enabled children to systematically work on the problem. Secondly, the teacher's contribution in (1) enabling and supporting the development of children's autonomy and (2) organizing the classroom in a way which gave the possibility for creative interaction to be developed. During the mathematical problem solving process, children demonstrated creative skills and abilities, such as fluency flexibility, originality, elaboration, persistence, patience and positive attitude towards error. Our findings also suggest that young children are able to use graphical representations in order to (a) detect original solutions, and (b) elaborate on existing solutions in order to detect all possible problem solutions.

Our findings are in accordance with other findings (Neumann, 2007; Kaufman and Sternberg, 2006; Haylock, 1987) underlining the important role of the teacher in setting and sustaining a creative environment. In our study, the classroom teacher played a central role in the development of a safe creative environment although throughout the process she was not the exclusive centre of the process. The children's expectations and interest were transferred to the experimental atmosphere created by the teaching situation, scaffolding their autonomy and providing multiple interaction experiences.

Lastly, in attempting to comment on the practical consequences of our findings, we could suggest that introducing young children to complicated mathematical problems within an environment of safety and encouragement could lead to the development of dynamic mental process including both, divergent and convergent thinking supporting them to rediscover mathematics acting as novice mathematicians.

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