Teaching the concept of angle through programming with Scratch

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ABSTRACT
The aim of this research is to cultivate geometrical thinking through the Scratch programming language. In particular, the research deals with the rotation as a dimension of the concept of the angle. This research includes the participation of 6th grade students who use Scratch to simulate the realistic situations given to them and that are related to the selected dimension of the angle. In this research, both the teaching model and the results of the activities from the use of the programming environment are presented. This pilot study shows that the first results from the implementation of the teaching model are encouraging.

KEYWORDS
Geometrical thinking, angle, rotation, simulation, Scratch

RÉSUMÉ
Le but de la recherche est de cultiver la pensée géométrique à travers le langage de programmation Scratch. En particulier, la recherche traite de la rotation en tant qu'aspect du concept d'angle. Dans l'enquête participant des élèves de la sixième classe (âgés de 12 ans), qui utilisent Scratch pour simuler des situations réalistes qui leur sont données et qui sont liées à l'aspect de l'angle choisi. La présente étude-pilote montre que les premiers résultats de l'application du modèle d'enseignement sont encourageants, car les étudiants sont capables de simuler des situations réelles par programmation visuelle et d'accéder intuitivement à la valeur arithmétique de l'angle. La recherche montre également que grâce à la programmation, les élèves peuvent surmonter les barrières cognitives telles que la portée de l'angle.

MOTS-CLÉS
Pensée géométrique, angle, rotation, simulation, Scratch

INTRODUCTION

The angle is a concept widely used in a variety of scientific fields. In school mathematics we find it mainly in Geometry, as well as in the field of Trigonometry. Since ancient times, mathematicians have tried to approach the angle concept and they have used it to provide solutions to geometric problems that were referred both at plane and space. However, the definition of the angle is so far a point of conflict among the researchers, since the angle is encountered in different application fields, from where its definition assumes different content. Let us note that Euclid, despite the systematic quoting of definitions in the concepts he uses, in his "Elements" he quotes the definition of the concept of angle, which is considered particularly problematic (Bunt, Jones & Bedient, 1988, p. 144).

According to Henderson and Taimina (2005), no formal definition of the concept of the angle can include all the aspects of our experience of what the angle is. Similarly, Mitchelmore and White (2000) report that there is a difficulty in defining the angle because each definition emphasizes the particular dimension of the angle to which it refers, but also there are different mathematical structures that require a different definition of the angle.

The present research presents the dimensions of the angle concept which have been approached by the researchers, as well as the categories of the notion of the angle. The aim of this research is to study the rotation as a dimension of the angle concept. The rotation will be approached through examples of everyday life, which will be designed by students through simulation by using visual programming. It is a pilot study where we will study the abilities of students to approach the angle through digital simulations.

THE CONCEPT OF THE ANGLE

Euclid in "Elements" defines the angle as "*a plane angle is the inclination to one another of two lines in a plane that meet one another and do not lie in a straight line*" (Bunt, Jones & Bedient, 1988, p. 144), although he does not define the notion of the inclination. Moreover, the above definition means practically, that he recognizes that there are angles the sides of which can be curved lines, something that he does not also use in "Elements".

Today, researchers propose different definitions of the angle. According to Devichi and Munier (2013) an angle can be defined in three different ways. Firstly, as a rotation angle, then as a sector angle (that is, the quantity that shares successive angular sectors) and finally as half lines extending from a common point (by opening or closing). Henderson and Taimina (2005) report that the angle can be considered as a geometric image, as a movement (dynamic meaning) and as a measurement. They refer that the first two aspects can be approached without the need for measurement. For example, we can tell a child to leave the door half-opened or open it widely without refer them the angle degrees.

Other researchers distinguish the dynamic and static concept of the angle (Close, 1982; Kieran, 1986; Scally, 1986). The dynamic concept refers to cases where the magnitude of the angle can be changeable, e.g. the opening of a scissor. The static concept refers to cases where it cannot be changed, e.g. the angle of a frame. Based on a more intuitive approach, Tanguay and Venant (2016) report that the most natural and comprehensible way of defining the concept of angle is to recognize that two rays starting at the same point divide the plane into two parts, each of which is defined as an angle. The one part is convex and the angle is called internal and the other part is non-convex which is called external, while as a special case, they recognize the straight angle which is formed by two opposing rays. Mitchelmore and White (1998), making a connection between the angle and everyday life, suggested a categorization of the angle in seven subcategories (rotation, meeting, inclination, corner, turning, direction, opening) each of which is associated with everyday experiences of the real world.

RESEARCH QUESTIONS

The present research aims to answer the following questions:

1. Can students simulate digitally the movement of the objects of their daily life, emphasizing their rotation angle?

2. How do they link the arithmetic value of the angle to the particular characteristics of each rotation state (rotation, number of full rotations, maximum rotation, axis of rotation)?

METHODOLOGY

Sample

In the survey participated five 6th grade students (11 years old) of a school located in a large provincial city in Greece. These students had been taught the angle from the formal curriculum and they were attending a formal class. At this level, students are taught to compare and measure angles, add and remove angles, and design angles using a protractor. It should be noted that they had not been taught in the past mathematics and the concept of angle using computers and this experience consists their first contact with the combination of mathematics and computers. The students were chosen because at this level the students are taught the angle as a basic element of the geometric and real world. We also believe that these students will be more ready to handle Scratch without previous experience with it, since they have received basic computer lessons within the ICT curriculum.

Material

For the implementation of the research, was used the categorization of the angle according to Mitchelmore and White (1998) since this categorization links the angle with the real world and at the same time provides the opportunity for the digital simulation of everyday situations by the students themselves. Of the 7 categories that have been proposed (rotation, meeting, inclination, corner, turning, direction, opening), in this research, it is presented only the first category, that is the actual or imaginary rotation around a certain axis or point. According to the same researchers, this categorization can include rotation situations from the daily life of students that are either unlimited or limited. For research needs, were designed 4 digital rotational situations from everyday life. Table 1 presents the particular features of each situation.

TABLE 1
Features of the angle situations

Angle Situation	Possibility of Rotation	Number of full Rotations	Maximum Rotation	Axis of Rotation
1: Fan	Unlimited	infinite	$> 360^0$	In the center
2: Door handle	Limited	zero	$\leq 90^0$	At the edge
3: Door key	Limited	Small number	$> 360^0$	In the center
4: Screw	Limited	Large number	$> 360^0$	In the center

The angle situations which were designed with the Scratch 2.0 language have a semi-structured form and are digital microworlds. Thus, they can be described as half-baked microworlds (Kynigos & Moustaki, 2014). In these microworlds, students can freely choose the code commands they need in each situation in order to move each object in a way that resembles the real experience.

Procedure

Since the students did not have previous programming experience, initially 5 teaching hours were spent in order to learn the basic functions of the Scratch 2.0 programming language environment. Then, the students processed individually the semi-structured microworlds and simulated the movement of the objects of each situation. The students, through the test and debugging process, corrected the code commands to manipulate the objects until they managed to simulate their movement as best as possible. After completing the simulations, they were given a worksheet in which they were asked to record similarities and differences of the four situations they handled. Finally, they were asked to report some of their own examples to match the situations they simulated. The final movement of the objects, the correctness of the code commands used by the students and the worksheet responses were evaluated. The duration of the research was 10 teaching hours. Overall, the procedure is presented in Table 2.

TABLE 2

Process of the implementation of the research plan

Phase	Activity	Duration
Teaching Scratch programming language	Edit objects and backgrounds - motions - views - pen designs - events - control	5 hours
Simulation development	Fan motion simulation	1 hour
	Door handle motion simulation	1 hour
	Door key motion simulation	1 hour
	Screw motion simulation	1 hour
Summary	Find similarities and differences – Find examples	1 hour

RESULTS

The results after the analysis of the code developed by the students for each angle situation are presented in Table 3. Each student's effort was labeled as “Correct” when the code commands were correct and the simulation was similar to realistic movement. When the code commands were partly correct but did not simulate the realistic movement well enough, they were labeled as “Incomplete” and when the code commands were wrong and unrelated to realistic movement they were labeled as “Incorrect”.

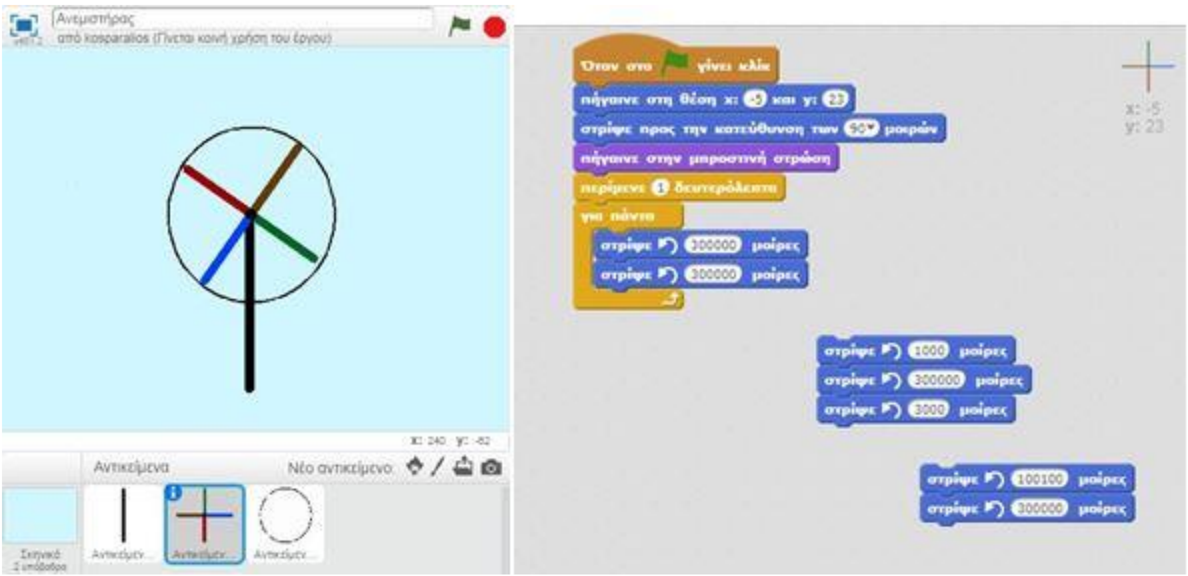
The first simulation that the students had to create was the fan. The students initially realized that they should use the “turn” command without showing any interest in the numerical value of the degrees of rotation. Then, through the test and debugging process, they realized that by increasing the numerical value of the degrees the rotation increases. Subsequently, all students found that as much as increasing the numerical value of the degrees, the fan is statically rotated and their model does not present the uninterrupted rotation. Significant is the fact that they used very large numerical values, for example 10,000 degrees. The 3 of the 5 students correctly constructed the simulation using the command “repeat forever” or “repeat for several times”. An example of such a solution is shown in Picture 1. These students observed that by increasing the numerical value of degrees, the fan moves faster by continuing its uninterrupted rotation. In contrast, 2 of the 5 students simulated incompletely their model, because although they used the correct rotation code commands, they did not show the realistic and constant rotation of the fan.

TABLE 3

Students' performance in individual angle situations

Angle Situation	Degree of Simulation Success	Number of Students
Fan	Correct	3
	Incomplete	2
	Incorrect	0
Door handle	Correct	5
	Incomplete	0
	Incorrect	0
Door key	Correct	5
	Incomplete	0
	Incorrect	0
Screw	Correct	5
	Incomplete	0
	Incorrect	0

FIGURE 1



Example of simulating the rotation of the fan

The simulation of the door handle movement was less difficult for students as all of them created a correct simulation of its movement. Students very quickly used the "turn" commands even if they were initially troubled about whether the door handle turns right or left. The analysis of the code commands used by the students showed that they placed sequentially the one command after the other without using the command "repeat", as it seemed they did not need it because of the limited rotation they had to present. All of the students used small values of degrees and did not exceed 90^0 in sum. In this way, they were able to show correctly the door handle movement without exceeding the vertical position.

The results are also encouraging for the third angle situation. All students were able to create correct simulations for the key door movement. They used commands exclusively sequentially one after the other without using the command “repeat”. They also used small values in degrees of rotation as they helped them to control the result through the small key movement. So, all the students tried to make simulation where the rotation of the key was not infinite and they gave importance to its limited rotation, as they have seen in their daily lives.

In the last simulation, the 5 students managed to select the right code commands to represent the movement of the screw. The 3 students maintained the same numerical value of the degrees of rotation (e.g. 15 degrees) and used the “repeat” command for a few times. The 2 students maintained the same numerical value of the degrees and sequentially set the rotation commands one after the other. All the students took advantage of the real life experience and paid attention to show that the screw rotates more than once, but not infinitely, as they said it is probably not screwed very well when rotated less than once.

CONCLUSIONS

The aim of the research was to find out the possibility for students to simulate, through visual programming, angle situations from their everyday life, emphasizing the angle of rotation. Also, the research attempted to study how pupils associate the arithmetic value of the angle with the particular characteristics of each angle situation. Moreover, we were not interested in checking the direction of rotation but the dynamic manipulation of the angle and the ability to understand that there are angles greater than 360^0.

Thus, the research showed that students have the ability to create their own models that represent objects and situations of the real world and have the ability to simulate their movement through visual programming. The ability of students to improve their simulations was increased from activity to activity and they handled angle situations in greater detail. They also completed faster each new activity given to them.

They used the arithmetic value of each angle very flexibly because they were not limited to numbers up to 180^0 or 360^0. This finding shows that the use of visual programming to create simulations can make students overcome misunderstandings that other researchers have observed (Devichi & Munier, 2013; Bütüner & Filiz, 2017), according to which students have difficulty in recognizing as angles 0^0, 180^0 or 360^0. Also, the research showed that the arithmetic values of the angle chosen by students were intuitively linked to each angle situation, but in a way that takes into account the individual characteristics of each situation (rotation, number of full rotations, maximum rotation, axle positioning rotation). This gave them the opportunity to suggest several of their own daily examples that resemble the movement of the objects they were given to handle (e.g. fan: helix, windmill, wheel, hamster wheel, etc.).

In conclusion, the process of simulating the movement of everyday objects was very pleasant and creative for students. They recognized the angle and the numerical value as the main element for the successful completion of activities. Moreover, the use of programming helped students overcome cognitive misconceptions related to the range of values that an angle can take. The findings concern only the small sample of the students who participated and the particular class they were attending. In the next stage, the research will be carried out in a larger sample of students and will include aspects of the angle that are not included in the present study in order to ascertain the students' ability to simulate digitally a larger variety of angular situations.

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