The development of practices for measuring length in preschool education

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Abstract
Could measurement of length be the object of teaching in early education? Could appropriate teaching interventions develop as to scaffold children's efforts to measure lengths? These are the central questions of this paper. The sample consisted of 84 children of six public preschool classrooms and the teaching intervention included two different kinds of tasks. Direct comparisons with the use of practices of one magnitude covering another or placing one next to the other and indirect comparisons by using tools that mediates in the measurement. The results of our research showed that practices for measuring length can be introduced into teaching as early as preschool education.

Keywords

Mathematics education - Measurement length - Preschool education

The concept of length and its measurement

Despite the emphasis placed by curricula on the familiarization of pupils of preschool and early school education with the practices of measuring length, studies show a low level of comprehension of the logical principles that are incorporated in these practices (Kamii & Clark 1997; Boulton-Lewis, 1987; Nunes et al. 1993). The studies in question show that young pupils often comprehend neither the necessity of introducing a unit of measurement, not the rules that govern its use. This lag is blamed mainly on the way of teaching, which emphasizes measurement techniques without accompanying them with the respective frameworks that give meaning to the measurement processes (Boulton-Lewis, 1987; Boulton-Lewis et al. 1996; Kamii & Clark 1997; Nunes et al. 1993; Zacharos, 2006; Zacharos et al., 2011).

Length measurement is a typical case of linear measuring, which is offered as a teaching object to preschoolers because measuring practices can be more easily understood than the measuring of other magnitudes such as, for example, area, volume or weight.

Studies marked by a social-historical perspective place special emphasis on the role of the tools that mediate and facilitate measurement (Clements & Stephan 2004; Clements & Sarama 2009; Nunes et al. 1993; Sarama, et al., 2011). They claim that the involvement in measurement processes can contribute to early capabilities, in terms both of constructing tools for the measurement of length and of using them in a variety of cases of measurement. In addition, the introduction of a measuring tool mediates and supports the acquisition of new knowledge.

Direct and indirect comparison of length

In the initial stages of acquiring skills in length measurement, children make direct comparisons using of practices of one magnitude covering another or placing one next to the other. In cases of direct comparison, children perform "calculations" of a qualitative nature that are conceptually based on the principle that two objects under comparison are equal in length when their ends correspond.

In a subsequent development phase of the ability to measure length, the child begins to make indirect comparisons by using an object, the length of which is similar to the length of the two lengths under comparison. This object is used as an intermediate tool. In more complex cases where measurement is required, the tool/measure that mediates in the measuring partitions, through its successive repetition, the magnitude being measured. The times that the unit of measurement fits inside the magnitude being measured gives the numerical result of the measurement. More precisely, the case of length measurement involves concepts and processes such as (Clements & Stephan, 2004; Kamii & Clark, 1997): Selecting the unit of measurement (unitization), which will be used to unitize the magnitude to be measured; The partition of the magnitude under measurement into distinct parts, which it is the mental cutting up of the length we wish to measure into equal units; Unit iteration in the magnitude under measurement; Transitivity reasoning, which makes possible indirect comparisons; The conservation of length as it is described in Piaget's theory (Piaget et al., 1960), according to which the length of an object does not change when it is positioned differently in space; The accumulation of distance, which refers to the understanding of the fact that the part which is covered by, say, two units, is contained within the part that is covered by three; The correlation of measurement and natural numbers. In this way, the continuous magnitude of length is transformed through the use of natural numbers and thus takes on a distinct existence (Newman & Berger, 1984).

The Greek kindergarten curriculum suggests that the teacher should implement teaching practices that enhance the scientific thinking of students. More specifically, the kindergarten curriculum states that to promote this objective, specific educational processes should be selected associated with the active participation of students in constructing knowledge. These procedures refer to controlling and testing hypotheses, data processing and formulation of appropriate questions to enhance children's interest for scientific matters.

This study aims to offer empirical material towards the further elaboration and development of a Greek preschool curriculum for teaching practices for measuring geometrical magnitudes, such as length. The research questions investigated in this paper are:

- Can pre-school students carry out activities such as comparing and measuring lengths?
- Can teaching contribute to the development of children's ability to compare and measure lengths, through proper planning of activities?

Methodology

The research was conducted in six classes of public kindergartens in Greece (a total of 84 children, approximately 5 years old). Teaching was done by the teachers who already were teaching in these classes and had attended a seminar on the content and the objectives of teaching. The survey included an introductory assessment of children's knowledge, which took place during individual interviews by the teacher of each class. The teaching interventions were made on groups of four or five children, created after the introductory assessment process. Children worked in small groups which allowed them to use the material and measurement tools, ensured greater uniformity in the conditions of children's interaction with the material and allowed better monitoring of the measurement strategies developed by the students. Nineteen student groups were created and each group had a separate unit of analysis.

The data collection was made by recording the verbal exchanges of students in each group and by using the comments of researchers into an observation protocol. In the part of the research which will be presented here, we are dealing with the part of the curriculum related to the indirect comparison of lengths.

The results of the study

In the analysis of the data presented here we will place special emphasis on the findings that emerged during the teaching. More precisely, we will concern ourselves with the ability of the pupils to respond to the measurement tasks that they were given, the strategies they followed in comparing lengths, as well as the improvement of measurement practices during teaching. What's more, we will observe the forms of interaction among the children and between the children and the teacher.

Indirect comparison. The use of different measurement units in sufficient quantity.

The activities presented here are two. Both have as main objective the introduction of a measurement unit in order to compare the length of two magnitudes and also to highlight the importance of using a single unit of measurement for all individual measurement procedures

First activity

Under the proposed scenario the students had to choose between two paths that were leading to a castle, the shortest one. On a three-dimensional model the two routes were designed leading to the entrance of the castle. The lengths of the two routes differed by little and were placed in different perspectives in order to avoid a direct estimation of the length which would be based on vision. To measure the routes, there were provided sticks, 6 cm and 8 cm long, in a sufficient amount to cover the distance. The teacher introduced the teaching situation to the children by using questions like "Which is the shortest way to the castle?", or in some cases when the route had to be indicated "Why do you think this route is the shortest one?" or "Why are you sure that this route is the shortest?" With the proposed teaching condition we wanted to draw the attention of children in the measurement unit selected each time.

*Measurement strategies***.** The majority of the students (sixteen groups of students) seemed to be familiar with the covering processes developed in earlier forms of direct comparison and they used covering strategies on the "routes" using the units that were given to them. In the case of the three groups that the “covering” practices were not used, the teacher asked the children to use the "tools" (sticks-units) that were given as a help.
Students used a great variety of measurement methods. It was observed that children often (seven groups of students) have used different units of measurement for each of the two routes. Also, different measurements tools were used even for the covering of the same route (in four groups of students). This led to very different conclusions about the length of the routes. In these cases the teacher intervened to lead children’s focus on the measurement unit. The issue of selecting the measurement unit was discussed, aiming to a common decision upon which a strategy had to be followed. The kindergarten teacher was seeking to destabilize the misconceptions of children and through practices that had the characteristics of socio-cognitive interaction (Doise & Mugny, 1981) to adopt scientifically sound measurement practices, as in the case of the student groups in Extract 1 below.

Extract 1. Destabilising misconceptions

S6 (Subject): We will put these (smaller units) here (along one side) because this route is shorter and these, (bigger units) there, because that route is longer.
T (Teacher): How many of these sticks (showing the big unit) did you put on that route (the «longer» route)?
S6: Seven.
T: And how many of the small ones on the other route?
S5: Eight.
T: So, which route is longer?
Here we have all the disagreements in the group. The teacher places one unit next to the other and draws their attention on the different size of every stick.
T: These sticks, which are our measurements units,, are they equal?
Subjects: No!
T: Now that we have seen that our units are not equal, should we try to find which route is the longest in a different way?
S5: Let’s put the same sticks on both routes.
T: Do you all agree that we have to use the same sticks?
The children accepted the use of a common unit in both measurements, they measured and finally the decided on the longest route.

Three groups of students have used units of the same length for the covering of the two routes. Then, counted and compared. Also in three other groups, students covered one route using only units of the same size and then placed the same units on the other route. Depending on whether the units were sufficient or not to cover the second route, they concluded on the length of the route respectively. The strategy chosen by certain pupils show a high level of ownership of transitivity reasoning, which is inherent in indirect measurements. Finally, two groups used the iterative use of a unit in a consistent way: Every time they used the unit, they placed their finger to mark the end of the measurement and continued from this point on.

Second activity

The emphasis on the numerical results of the measurement, without the parallel emphasis on the unit used, is often giving numbers that are "free" from any natural content. This leads to inconsistent measurements and inaccurate conclusions when units of different lengths are used to measure the same length or to compare different lengths.

The second activity, that supports the first one, intended to draw students attention to the relation between the outcome of a measurement and the measurement unit used each time. According to the scenario that came with the activity, the central character of the story had to cross a river but the wooden (straight) bridge was broken. Children had to give instructions to the builder about the length of the bridge, in order to construct a new one. As materials for the activity, two different measurement units (same as previous activity) were given, in sufficient quantity to cover the length.

The conclusion of the previous activity on the use of the same unit in each individual measurement is noted in the beginning of the activity. The pre-school teacher introduced to the children the teaching situation using the following questions: "What instructions will you give the builder to build the bridge? How long will it be?" "Can we measure the length of the bridge?", "How will we measure it? Do you have any idea? ".

The groups communicated their results. In the case when different units of measurement were used and therefore we had different outcomes, the teacher asked the children to justify the different results with questions like: "Is any of the bridges larger than the other?", "(Speaking in each group) What measurement units did you use? Are your units of the same size? ". If all teams used the same measurement units, the teacher asks: "If we measure using this (pointing to another unit) we will need more of these sticks (units) or less? Why?", "So, what instructions will we give to the builder?"

Measurement Strategies. As children were familiar with the procedures of indirect measurement of the previous activity, they used mostly the same measurement unit (twelve groups of students).

There were cases (five groups of students) where the disagreements of the group members on which unit to be selected, led to the use of both units. This was done by two parallel measurements along the "sides" of the bridge and on one occasion in two consecutive measurements. The "obligation" of students to give instructions to the builder, was forcing them to seek for answers that they all agree, like in the next dialogue, where group students measured using both measurement units (Extract 2).

Extract 2. An agreement on a commonly acceptable unit

T: What we will say to the builder? How long is the bridge?
Students are giving different numerical results
T: How many are they? (Showing the bigger sticks)
Subjects: Four.
T: And these? (Showing the smaller sticks which are placed on the other side of the bridge)
Subjects: Six.
T: So, what are the instructions that we should give to the builder? What will we say to him? Four sticks of these or six of these? (Showing the corresponding sticks)
The children finally agreed on using the bigger unit for the measurement.

Similarly, in another group, the students measured both "sides" of the bridge by placing different measurement units on each side. The pre-school teacher was intervening in order to draw students' attention on the difference between the size of the units. Her dialogue with the students, (Extract 3) shows the development of the ability of these students to relate the numbers obtained from the measurement with the size of the unit.

Extract 3. Relation between numerical expression and physical size of the unit

T: We have to say to the builder how long is the bridge.
S23: Let's measure it.
T: Great idea. How do we do that?
S23: Places the small sticks along one side of the bridge, while another student (S25) places the longer sticks on the other side.
T: Now that you have placed the sticks, how long we will say to the builder that the bridge is?
Students are giving different answers.
T: So, what do we say to the builder?
S23: We will say 6 sticks long (showing the sticks he has been using).
T: You S25, what do you think;
S25: Will say five (showing the longer units).
T: Five what;
S25: Five sticks like these (showing the sticks).
T: Very good! In order not to confuse the builder with different numbers, should we agree on say the same number? What do you think?
S24: Let's say six (small sticks) and we can give him one of these so that he will understand.

When groups finished the measurement process, the choice of the measurement unit was discussed among all groups of pupils and the numerical results were written on the board. The fact that the "builder" had to understand what do the numbers "4" and "6" mean to complete the order, led to the design display of each unit next to their respective number.

Indirect comparison. Repetitive use of the unit

The purpose of the next activity was to develop a more complicated level of the measurement capability development, based on the repeated use of the unit (Barrett et al., 2011).

In previous activities many copies of each unit were distributed so that the units would be sufficient to cover the measured length. The measurement, however, became more difficult when a single unit was given and the students had to devise the repetitive use of the unit on the length and therefore to devise ways to capture the "marks" left by the measure while used repetitively.

Here, under this scenario, the children must help the character of the story to choose the lower of the three windows of a tower and give instructions for building a staircase up to the height of the window. The "windows" were displayed on different sides of the kindergarten classroom, so there was no possibility of visual comparison of height. Their height was measured using the units of 6 cm and corresponded to numbers 7, 8 and 9. Students were given only one unit (one stick of 6cm) and a pencil to facilitate their notes.
Also, in this activity, the questions of the kindergarten teacher facilitated the integration of children in the teaching situation: "Which window is at a lower height? Why?", "Can you help the prince to find how tall the staircase that he should build must be?”

Measurement strategies. In some cases children (four groups) did not use the materials given to them and painted stairs from the floor to the windows. In many cases (seven groups) students were trying to count how many times the unit fits in the distance, but that was inconsistent and without marking the spot where the unit reached each time. In another case a student placed the unit and designed the outline trying to visualize the steps of the "ladder". The student realized that this process was time consuming and the teacher encouraged him to devise a more effective way. Indeed, the student repeated the test with the "dominant" dimension of the unit (extract 4)

Extract 4. Emphasizing on the “dominant” dimension of a unit.
S45: Can I place this (the unit) like this so that it can “climb up” (turns the unit vertically and designs the outline of the stick
T: And now, how will you proceed?
S45: We will put it (the stick) on top
He continues the process.
When the student finished, he counted all the "marks" of the unit, he designed and "ordered" a ladder with 8 steps, as many as the units used.

In another case the teacher with her interventions sought to draw students' attention to the materials given (the stick and the pencil). Here the pedagogical framework and especially the directive that required to provide instructions for the construction of stairs, led the students to disengage from a direct sensory experience and to develop metacognitive type approaches (Extract 5).

Extract 5. Familiarization with the measurement process
T: We have to give to the builder correct instructions. So, we have to count right.
S7: To build a tall ladder.
T: How tall will that ladder be?
S7: That tall (shows the height with the finger!
T: Yes, but the builder cannot see you and we have to give him the correct instructions.
S9: Let’s measure it then.
T: Go ahead, tell us (refers to S9).
S9: The first step will be there (marks the point where the unit ends). Here we will make a step (marks with the pencil) and then another one (marks)...
The student completed the process and then all together counted the "steps". The process was repeated for the other windows.
T: So, what instructions will we give to the builder?
S7: To make a ladder with seven steps.
T: And how big will the steps be?
S6: That big (showing the unit).
S7: Let’s draw on a paper the stick (the unit) to give it to the builder so that he will know how tall he must make the ladder.

Forms of interaction

In the protocol of observation there were also recorded forms of interaction developed during teaching. The teachers created a laboratory atmosphere among the groups of students. The students could handle the materials and experiment by trying and proposing various solutions. They encouraged the communication among students, both within each group and between groups and with appropriate verbal interventions, enhanced the development of reasoning in children. Also, inner-group discussions were developed and sometimes disagreements. The pedagogical framework, the scenarios that came with the activities, helped to reach common agreements.

Discussion

The results of our research showed that practices for measuring length can be introduced into teaching as early as preschool education.

Indeed, the teaching situations proposed and the interventions of the teacher showed that the pupils that made up the sample responded satisfactorily to tasks of comparing and measuring lengths. We saw that, either of their own accord or following suggestions by the teacher, the children devised ways of comparing lengths, such as using the unit of measurement. In this case, the unit of measurement was used in two ways: through the strategy of covering, whereby the children use the available units of measurement to cover the lengths and count the units used for each measurement; or though the repeated use of the single unit of measurement.

Through our research we have tried to highlight the major role of activities carried out within a playful structure in the development of forms of mathematical thought. Occupying a child with mathematical concepts within the framework of a game has a functional nature and is richly signified, a fact that reinforces the redesigning and enriching of educational practices with forms of teaching that are based on the introducing of suitable activities
In these cases, the measuring tool becomes meaningful to the children, who acknowledge its significance, as, for example, in the case in which they had to choose the shortest route.

While the use of the unit of measurement enables the quantification of length and, hence, the precise measuring and correlating of the size of two or more magnitudes, in a child’s mind, numerical expression is often disengaged from the unit of measurement as a physical magnitude (Zacharos, 2006). Consequently, the number that results from the act of measuring is seen as an absolute magnitude which is unrelated to the size of the unit used each time. The cognitive conflict created by the concurrent numerical equality and inequality of the same magnitudes gives rise to a discussion, which, in the end, leads to a concurrence regarding the need to use a fixed unit for each act of measuring.

In conclusion, in our view, preschoolers should be encouraged to undertake measurement tasks, since they appear able to carry them out satisfactorily. With this paper, we wish precisely to underscore the need to enrich mathematical education in Greek preschool education through a cohesive curriculum which is related to the process of measuring geometrical magnitudes.

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