# Forms of argument used by pre-school children

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**ABSTRACT**
*This paper investigates five and six-year old children's ability to employ logical reasoning in their in-class argumentation. Thirteen pre-school children participated in the study. The children were involved in organized dialogues in order to investigate their ability to construct logical arguments. A simplified version of Toulmin's argumentation scheme (including only data, claim and warrant) was adopted in order to analyse the students' reasoning and to identify the different types of argumentation. Our findings suggested that the structure of the children's arguments accords with the reasoning found in formal logic, including inductive reasoning, as well as reasoning based on the law of excluded middle.*

**KEYWORDS**
*Pre-school, argumentation, reasoning, Toulmin*

**RÉSUMÉ**
*L'étude présente examine la possibilité d'enfants d'âge préscolaire (de cinq et six ans) d'utiliser des arguments logiques au cours de l'enseignement en classe. Treize enfants d'une école maternelle ont participé à l'étude. Les enfants ont été impliqués dans des dialogues organisés afin d'étudier leur capacité à construire des arguments logiques. Une version simplifiée du schéma d'argumentation de Toulmin a été adoptée afin d'analyser le raisonnement des élèves et d'identifier les différents types d'argumentation. Les résultats ont montré que les élèves sont en mesure d'utiliser des formes d'arguments trouvées dans la science de la logique.*

**MOTS-CLÉS**
*École maternelle, argumentation, raisonnement, Toulmin*

## INTRODUCTION

Amongst the goals of the contemporary curricula, as early as pre-school education, is the development of the children's creative and critical thinking (Van de Walle, 2007; Shiakalli & Zacharos, 2014).

The purpose of this paper is to investigate young children's, aged five to six year olds, ability to use forms of argumentation and specifically forms of reasoning that are identified in the science of logic; such as, deductive reasoning, forms of indirect proof, cause-and-effect relationships, or arguments based on inductive generalisations.

One of the aims of formal education is to facilitate the students' transition from informal forms of knowledge and the everyday use of language towards the use of scientifically acceptable forms of knowledge and language, including scientifically accepted and formulated arguments.

Critical thinking development allows people to seek and construct logically founded meanings, to justify their thoughts with logically based arguments and to convince themselves and persuade others with arguments founded on logic (rather than employing other forms of justification, including authority or affect-based claims). Hence, critical thinking allows people to evaluate their own practices on the one hand, and to agree or disagree with others' opinions justifying their choice, on the other. Consequently, critical thinking essentially concerns those crucial aspects of the educational process that enhance the development of metacognitive thinking strategies and facilitating the students' ability to reflect on their own learning processes (Fisher, 2007).

The teachers' role is important in children's critical thinking development since they are responsible for the designing of a learning environment, which allows for the young learners' formulating claims, questions, controversies and synthetic reasoning.

## FORMS OF ARGUMENTATION

Argumentation and reasoning are forms of logical thinking. Argumentation gives us an idea about the reasoning adopted by the students. Both notions in the spoken and written language consist of isolated, related or interrelated propositions, each having a different degree of generality.

In this section, we briefly discuss arguments in the form of deductive reasoning, as well as arguments based on non-deductive reasoning (Getmanova, 1989; Genesereth & Kao, 2013).

According to Aristotle, *deductive reasoning*, in their simplest form, consist of two premises and a conclusion, structured as follows: all p are q (first premise); t is a constituent of p (second premise); therefore, t is q (the conclusion). Deductive reasoning is characterised by its absolute degree of certainty, in the sense that given a specific case of a broader category and the rules of logic, what holds true for the category must hold true for the special case.

A type of non-deductive reasoning, called *abductive reasoning*, has the following structure: If p then q, q (true), then p (probably). Abduction essentially describes the process of conjecturing the premises from which a given conclusion may derive. Thus, abduction is linked with a smaller than one degree of certainty.

Another type of reasoning, which in formal logic is named as *modus tollens*, is based on the principle identified as the *law of excluded middle*. Arguments based on this reasoning, are structured as follows: if p is true, then q is true (first premise), but if q is not true (second premise), then p is not true either (conclusion). This type of reasoning is based on an internal structure and the conclusion arises as a unique and unambiguous consequence of the two

introductory statements (premises) of claims of the two premises. The relationship between premises and conclusion resembles the relationship between cause-and-effect, since "[t]he 'if' part explains the 'then' part" (Grabiner, 2012, p. 163).

The r*eductio ad absurdum* rule (or "the rule of introduction of negation") is another type of reasoning, indicating that a judgement *p* should be negated (considered necessarily false) if a contradiction derives from *p* (Getmanova, 1989).

Finally, *inductive reasoning* is the basis of arguments structured as: 'If $p_1$ then q', and 'If $p_2$ then q', and …, 'If $p_n$ then q', then 'If p then q' (where $p_i$ are n instances of the category p). It follows that the validity of an inductive reasoning derives from the number of instances identified as a premise. Hence, the degree of certainty of an inductive reasoning can be one, only in the case that all instances are considered, thus implying that usually an inductive reasoning is linked with a smaller than one degree of certainty.

## PRESCHOOLER'S ABILITY TO USE ARGUMENTS

Various studies have shown that four and five year old children, on the one hand, demonstrate the ability to employ deductive reasoning (Dias & Harris, 1988, 1990; Richards & Sanderson, 1999; Pournantzi, Zacharos & Shiakalli, 2015) and, on the other, they can evaluate the truth or untruth of a sentence, recalling arguments made in a previous time (Koenig, Clément & Harris, 2004). Often the children's answers were justified and based on the information provided by the premises (Dias & Harris, 1990).

Furthermore, four and five-year old children succeeded in the case of inductive reasoning, when asked to think within situations contradicting reality (German & Nichols, 2003). Nevertheless, the children face difficulties to deal with a long chain of sentences constructing the reasoning process, thus making it difficult for the children to follow the sequence of events (Beck, Robinson, Carroll & Apperly, 2006).

It should be noted that the aforementioned research projects were conducted within a pedagogical framework which assisted the children to accept the hypothetical situations and, thus, to succeed in the reasoning process activities (Dias & Harris 1990; Richards & Sanderson, 1999; Pournantzi et al., 2015).

Moreover, at the early stages of schooling, the argumentation and reasoning process development are solely based on the classroom forms of verbal interaction, since it is through such interaction that a common framework of meaning is established (Mercer, 1995; Storm, Kemeny, Lehrer & Forman, 2001; Mercer & Sams, 2006).

## METHODOLOGY

### *Sample*

This paper presents a case study carried out in April 2014 at a public pre-school setting in Greece. The sample consisted of thirteen children of a middle class social background (referred to as S1-S13 in this paper): nine boys and four girls (mean age 5.5). All participants had attended at least one year of formal pre-school and were familiar with adult-child teaching interactions. None of the children had been involved in forms of formal reasoning processes or argumentation within the school context.

### *Research design*

The study consisted of five autonomous teaching interventions. The pedagogical context for each intervention for the introduction and investigation of the children's reasoning process

was formulated by the reading of a different story. The stories were chosen by the research group based on their adequacy to serve the teaching goal. Story adequacy was established through a pilot study. All storybooks were published in Greek - either original works in the Greek language or works translated into Greek. Based on the original storybooks, five autonomous brief articulated stories were constructed in order to allow narration interruptions and to facilitate the children's discussion.

The duration of the intervention ranged from 25 to 40 minutes, depending on the story length and the children's interaction. The collected data included the audio recordings of the interventions and the researcher's field notes.

### *Analysis*

The structure of the childrens' argumentation was identified according to Toulmin's theory of argumentation (1958). In specific, we utilised a simplified version of Toulmin's model, consisting of three elements (see for example Pedemonte, 2008). The first step of an argument is expressed by a standpoint which is an assertion or an opinion. In Toulmin's terminology, the standpoint is called the *Claim* (C). The second step consists of the production of *Data* (D) supporting and justifying the claim. The third element, called *Warrant* (W), provides the justification for using the data; it provides support for the data–claim relationship. The warrant, which can be expressed by a principle or a rule, acts as a bridge between the data and the claim.

The structural analysis following Toulmin's model can be applied to all types of arguments (and proof in the case viewed as an argument) including the three common structures of inferences: deductive, abductive, and inductive (see Table 1).

**TABLE 1**

*Structural analysis of three types of arguments according to Toulmin's model*

| **Arguments** | **Type of arguments** |
|---|---|
| Deductive | Data → Claim<br>Warrant |
| Abduction | Data: ? → Claim : q<br>Warrant : p ⇒ q |
| Inductive | Data:<br>p1, p2, ...,pn<br>or<br>p1, p1⇒p2, p2⇒p3, ...<br>→ Claim : q<br>Warrant : Generalization |

From Toulmin's point of view, arguments are mainly individual rather than social acquisition while for Krummheuer (2007) perspective, arguments are socially motivated- he views them as the attempt of someone to persuade, using verbal expressions or external representations, for the correctness of his/her point of view.

## RESULTS

In this section we present selective data deriving from two teaching interventions, investigating the children's argumentation with respect to reasoning types discussed in the theoretical framework. In our selection, we focused on the method of children's argumentation analysis according to Toulmin's typology.

### *First teaching intervention*

*Pedagogical frame*: the teaching intervention was based on a story by Alemagna (2010) "A lion in Paris": *"A huge lonely lion felt extremely bored in the savannah so one day he left to find a job, some love, a future".*

*Extract 1. Cause and effect relation*

| | |
|---|---|
| *1.1* | *T (Teacher)*: What made the lion leave the savannah? |
| *1.2* | $S_3$: They didn't give him good food. He didn't like it. |
| *1.3* | *T*: He didn't like it. Why not? |
| *1.4* | $S_3$: Because… |
| *1.5* | *T*: Who didn't give him nice food? |
| *1.6* | $S_3$: The servants. |
| *1.7* | *T*: The servants. They didn't give him nice food. And why did he decide to leave? |
| *1.8* | $S_3$: Because he didn't like the food they gave him. |
| *1.9* | *T*: And he decided to leave? |
| *1.10* | $S_{11}$: he might not have liked the country. |
| *1.11* | *T*: he didn't like where he lived? And why do you think he didn't like it there? |
| *1.12* | What did he not like the most and he wanted to leave? |
| *1.13* | $S_{11}$: I don't know what he would have liked. |
| *1.14* | *T*: What did he not like? What made him wanting to leave? |
| *1.15* | $S_{11}$: …(Thinking) |
| *1.16* | *T*: What could he not like about his country? |
| *1.17* | $S_{11}$: He might not have liked the lake he had. |
| *1.18* | *T*: He might not have liked the lake he had. |
| *1.19* | $S_8$: He might not have liked the house, this country, food and all he had seen, because he saw what he had seen. |
| *1.20* | *T*: Oh! So you say he saw everything in his country. And? He decided to leave? |
| *1.21* | $S_8$: Yes! This is why he left. To see (*means to see other things).* |
| *1.22* | *T*: To see what? |
| *1.23* | $S_8$: To see a new country. How it is. |

In the above dialogues (lines 1.2, 1.10, 1.17, 1.19-1.20) subjects show a series of possible causes for the lion wanting to leave the savannah. According the Toulmin's scheme, children formulated argumentations based on inductive reasoning (Table 2).

**TABLE 2**
*Inductive reasoning*

| | | |
|---|---|---|
| **Data:**<br>$S_3$: Because they didn't give him nice food. He didn't like it.<br>$S_{11}$: He might not have liked this country.<br>$S_{11}$: He might not have liked the lake he had.<br>$S_8$: He might not have liked the house, this country, the food and all he had seen, because he saw what he had seen. | → **Warrant**: Generalization | **Claim:** The lion decided to leave the savannah. |

Story continuation: *"He got to Paris by train without any luggage. It was his first time in a large city. And, not to our surprise, he was a little scared."*

*Extract 2. Cause and effect relation*

| | |
|---|---|
| *2.1* | T: it was the lion's first time in a large city such as Paris and he was a little scared. |
| *2.2* | What scared him? |
| *2.3* | $S_4$: People. |
| *2.4* | T: People. What else? |
| *2.5* | $S_3$: The train. |
| *2.6* | T: Why did the train scare him? |
| *2.7* | $S_3$: Because they make a lot of noise and it might have deafened him. |
| *2.8* | T: What else could have scared him? |
| *2.9* | $S_5$: So many people. |
| *2.10* | T: So many people. What else? If we go into a large city, what can usually scare us? |
| *2.11* | $S_5$: The people might have been a little strange and he felt scared. |
| *2.12* | T: people might have been a little strange and he felt scared. What else could have scared us if we had gone into a strange city? |
| *2.13* | $S_3$: Animals. |
| *2.14* | T: Why would animals scare us? |
| *2.15* | $S_3$: Because strange animals, which we have not seen in our city might scare us. |
| *2.16* | T: Strange animals can scare us. What else could scare us? |
| *2.17* | $S_{12}$: Some people might be bad people. |
| *2.18* | T: What do you mean bad? |
| *2.19* | $S_{12}$: Wanting to kill us. |
| *2.20* | T: They wanted to kill the lion? |
| *2.21* | $S_{12}$: Yes! If they were hunters! |

In dialogues 2, children also develop arguments which are based on inductive reasoning as shown in Table 3.

**TABLE 3**
*Inductive reasoning*

| | | |
|---|---|---|
| **Data**:<br>S4: People.<br>$S_3$: The train. Because they make a lot of noise and it might have deafened him.<br>$S_5$: So many people. The people might have been a little strange and he felt scared.<br>$S_3$: Animals. Because strange animals, which we have not seen in our city might scare us.<br>$S_{12}$: Some people might be bad people. Wanting to kill it. | → **Warrant**: Generalization | **Claim**: The lion was scared in such a large city as Paris. |

***Second teaching intervention***

*Pedagogical frame:* the teaching intervention was based on the story "Shovel on Mars" (Trivizas, 2013), edited by the research team for the needs of the study: "Once upon a time, three astronauts went to explore Mars. When they finished exploring the planet they decided to go back to Earth. They collected all their things and left. But they forgot to take their shovel. When the astronauts left, the aliens on Mars, called Martians, walked around the strange object they had never seen before and kept asking one another 'what is this thing?'. After long talks and hard thought they decided that it was a lamp post".

***Modus tollens and inductive reasoning***

*Extract 3. Modus tollens reason and inductive reasoning*

| | |
|---|---|
| *3.1* | *T*: if this was a lamp post (shows the shovel) would it have something special? |
| *3.2* | $S_2$ and *S3*: Light. |
| *3.3* | *T*: Light. But is this (shows the shovel) a lamp post? |
| *3.4* | *Students*: No! |
| *3.5* | *T*: Why? |
| *3.6* | $S_4$: Because it has no light. And it does not look like that. |
| *3.7* | *T*: How so? Can you explain? |
| *3.8* | $S_4$: It has no light, it has nothing to switch off. |
| *3.9* | $S_{13}$: Because it has no wood at the top and no metal at the bottom to dig. […] |
| *3.10* | $S_{11}$: Because it has no lamp. […] |
| *3.11* | $S_9$: Because it has no electricity. […] |
| *3.12* | $S_6$: It has no cord. […] |
| *3.13* | $S_4$: It has no switch to turn on. |

In Extract 3, it is argued that the children's reasoning resembles the structure of modus tollens reasoning (lines 3.1-3.6). Nevertheless, the children's reasoning was incomplete in terms of formal logic, since the second premise was based on incomplete inductive reasoning, where the concept of the "middle term" did not consist of all possible cases (see Table 4).

**TABLE 4**
*Form of modus tollens reasoning*

| | **Reasoning structure** |
|---|---|
| 1st premise | If statement p is true then statements $q_1$, $q_2$, $q_3$, … are also true. |
| 2nd premise | But statements $q_1$, $q_2$, $q_3$, … are not true. |
| Conclusion | Then p is also no true. |

According the Toulmin's scheme of argument here has the form showing in Table 5.

**TABLE 5**
*Modus tollens reasoning*

| | | |
|---|---|---|
| **D***: if this was a lamp post* (p) | **W**: p→q | **C**: (It would have) Light (q) |
| Then, the p statement is false also | ← | But this statement is false<br>*S4*: Because it has no light. And it does not look like that.<br>*S4*: It has no light, it has nothing to switch off.<br>*S13*: Because it has no wood at the top and no metal at the bottom to dig. […]<br>*S11*: Because it has no lamp. […]<br>*S9*: Because it has no electricity. […]<br>*S6*: It has no cord. […]<br>*S4*: It has no switch to turn on. |

*Story continuation:* "The Martians continue looking at the strange object trying to find out what it is. "Might it be a lion?, said A Martian. "Yes, yes it is", said another Martian. "Just a moment! It does not look like a lion to me", said a third Martian".

*Extract 4. Modus tollens reasoning*

| | |
|---|---|
| *4.1* | *T*: If this (shows the shovel) was a lion, would it have something special? |
| *4.2* | *S4*: It would have a tail. |
| | […] |
| *4.3* | *S2*: It would have a mane and sharp teeth and it would eat them up. |
| | […] |
| *4.4* | *S13*: It would have eyes and sharp nails and a tongue. |
| *4.5* | *T*: Good! So has this (shows the shovel) got sharp teeth to chew? |
| *4.6* | *Students*: No! |
| *4.7* | *T*: Why? |
| *4.8* | *S4*: Because it is not a lion. It is a shovel. |

In Extract 4 children based again their justification on an incomplete modus tollens form of reasoning based on the law of excluded middle, where the concept of the "middle term" did not consist of all possible cases (Table 6).

In this case we again see the subjects contributing to the formulation of argument, more specifically the formulation of claim, giving possible effects from the assumption of the premise.

**TABLE 6**
*Modus tollens reasoning*

| | | |
|---|---|---|
| **D***: If this (shows the shovel) was a lion* (p) | **W**: p→q | **C**:<br>S4: It would have a tail.<br>$S_2$: It would have a mane and sharp teeth and it would eat them up.<br>$S_{13}$: It would have eyes and sharp nails and a tongue (q) |
| Then, the p statement is also false | ← | But, all these statements are false |

*Story continuation: "*The Martians continue to look at the strange object with the same curiosity. "Could it be a sponge?", said a Martian. "Yes, Yes. It is a sponge", said another Martian. "It does not look like a sponge to me", said a third Martian".

*Extract 5. Shovel and sponge comparison*

| | |
|---|---|
| *5.1* | *T*: Can this (show the shovel) be a sponge? |
| *5.2* | *Students*: No! |
| *5.3* | *T*: Can it not be a sponge? |
| *5.4* | $S_{12}$: Because if you squeeze it you will not get any soap and water. |
| *5.5* | $S_5$: And it is not square with soap and water and soft either. |

In Extract 5, children formulated argumentations based on inductive reasoning (Table 7).

**TABLE 7**
*Inductive reasoning*

| | | |
|---|---|---|
| **D***:*<br>$S_{12}$*: it... (has) not get any soap and water*.<br>$S_5$*: And it is not square with soap and water and soft either.* | **W**: Generalization | **C**:<br>*Students*: No! (it not be a sponge) |

## DISCUSSION

The findings of this study support the thesis posed by various researchers that through appropriate pedagogies it is possible to facilitate the pre-school children's developing of critical thinking within a pedagogical context guiding the employment of logical reasoning (Dias, & Harris, 1998, 1990; Richards, & Sanderson, 1999; German, & Nichols, 2003).

Various research projects have investigated the students' argumentation processes, suggesting the convergences and divergences of 'everyday' (for example, narrations, discussions) and formal argumentation, including primary education children, university students and mathematicians (Inglis, Mejia-Ramos & Simpson, 2007; Pedemonte, 2007; Weber, Maher, Powell, & Lee, 2008; Moutsios-Rentzos, 2009; Moutsios-Rentzos, & Simpson, 2011). The findings of this study complement and extend this line of research showing that the pre-schoolers were able to respond by employing lines of arguments

resembling the argumentation forms of the law of excluded middle and of inductive reasoning.

Not only did the children use the given information in order to come to the correct conclusion, but they also seemed to have accepted the commitments required by the activity communication frameworks, referring to situations contradicting their experiences. Moreover, students seemed to have developed argumentation structures that can be applied on hypothetical situations. The above observations were in agreement with other research findings (Beck et al., 2006) showing children's ability to cognitively decentralize from their immediate experiences, as well as their ability to build reasoning processes based on imaginary or hypothetical situations.

The communication framework within which such concepts were integrated appeared to stimulate children's interest and to determine their positive emotional attitude towards the specific aims. Here, the communication framework's significant parameters were the verbal interactions between children and adult.

As noted by Mercer (1995), the use of language is the means for creating meaning between teacher and student. Guiding students, through suitable questions, is critical in children's familiarization with the reasoning processes.

In the case of this study we see a development of Mercer's cumulative talk, which is characterized by the additive submission of arguments, repetitions and rephrases.

In conclusion, it is stressed that this was the first time, within the classroom context, that the children had come into contact with such cognitively demanding processes. The children appeared to be able to successfully cope with such situations, suggesting the possibility of introducing these concepts in pre-school education. We posit that the story-based teaching interventions can be applied across multiple disciplines including mathematics, science education, and linguistic education, complementing existing research projects that draw upon embodied experiences of phenomena (see for example, Moutsios-Rentzos, Spyrou & Peteinara, 2014), in the sense of moving the cognitive focus on thought experiments in a linguistically defined setting that may expand or transcend a merely perceptually defined phenomenon. Further research is currently designed and implemented with the purpose to investigate the systematic introduction of formal reasoning processes in young ages, focussing on appropriate teacher preparation and encouragement.

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